Departamento de Engenharia Mecânica
ENG 1704 - Mecânica dos Sólidos II

Fórmula da Secante para o Cálculo da Tensão Máxima em Barras Carregadas Axialmente

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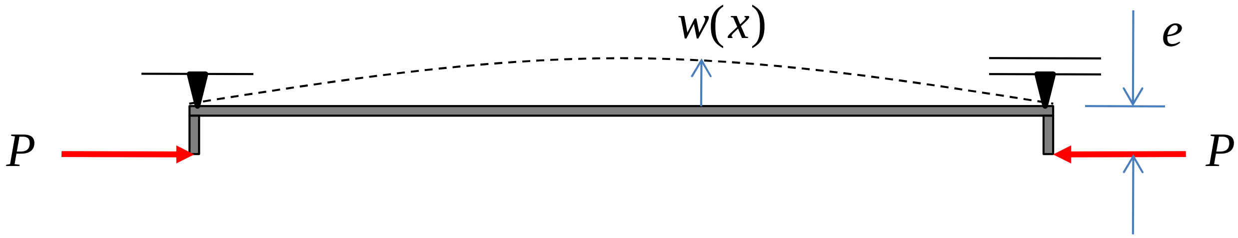
Referência: S. H. Crandall, N. C. Dahl, and T. J. Lardner, *An Introduction to The Mechanics of Solids, 2nd ed.*, McGraw-Hill, 1978



$$\left.\begin{aligned} &\frac{d^2w}{dx^2}+\frac{P}{EI}w=-\frac{P}{EI}e \\ &w(0)=0 \\ &w(L)=0 \end{aligned}\right\} \Rightarrow \quad w(x)=e\tan\frac{\lambda L}{2}\sin\lambda x+e(\cos\lambda x-1) \qquad \lambda^2=\frac{P}{EI}$$

$$\delta=\max\{w(x)\}=w(L/2)=e\left(\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)-1\right) \qquad P\to P_{cr}=\pi^2\frac{EI}{L^2} \quad \Rightarrow \quad \delta\to\infty$$

$$\max\{|M(x)|\}=M(L/2)=Pe\sec\left(\frac{L}{2}\sqrt{\frac{P}{EI}}\right)$$

$$\max\{|\sigma_{xx}(x)|\}=\frac{P}{A}+\frac{M_{\max}c}{I}$$

$$\sigma_{\max} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{EA}}\right)\right]$$

Definindo:

$$p = P/S_Y A$$

$$\varepsilon = ec/r^2$$

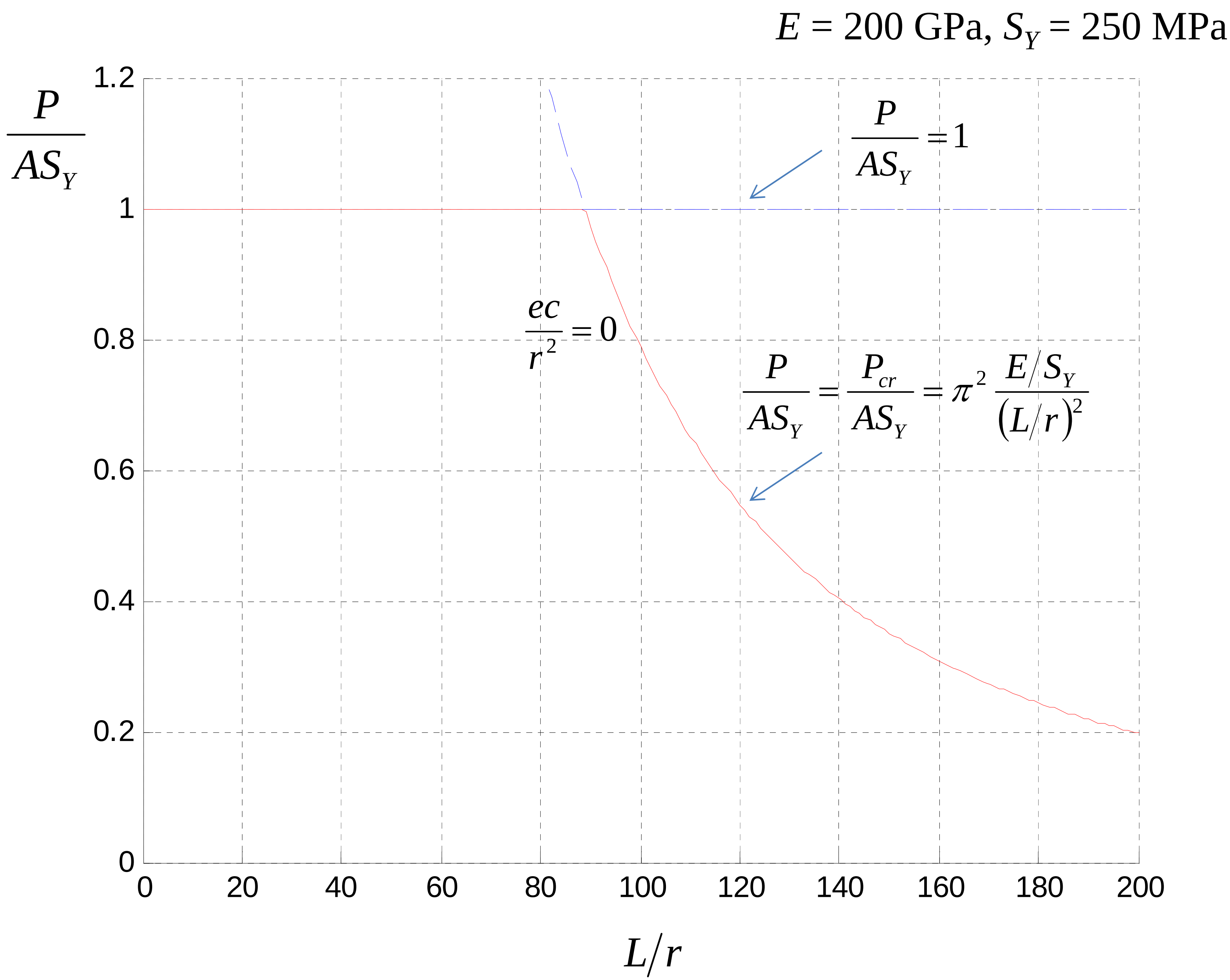
$$\xi = L/r$$

$$\alpha = E/S_Y$$

ε : Razão de Excentricidade

ξ : Razão de Esbeltez

$$\frac{\sigma_{\max}}{S_Y} = p\left[1 + \varepsilon\sec\left(\frac{1}{2}\xi\sqrt{\frac{p}{\alpha}}\right)\right]$$



E = 200 GPa, S_Y = 250 MPa
$\frac{P}{AS_Y}$
$\frac{P}{AS_Y} = 1$
$\frac{ec}{r^2} = 0$
$\frac{P}{AS_Y} = \frac{P_{cr}}{AS_Y} = \pi^2 \frac{E/S_Y}{(L/r)^2}$
1.2
1
0.8
0.6
0.4
0.2
0
0
20
40
60
80
100
120
140
160
180
200
L/r

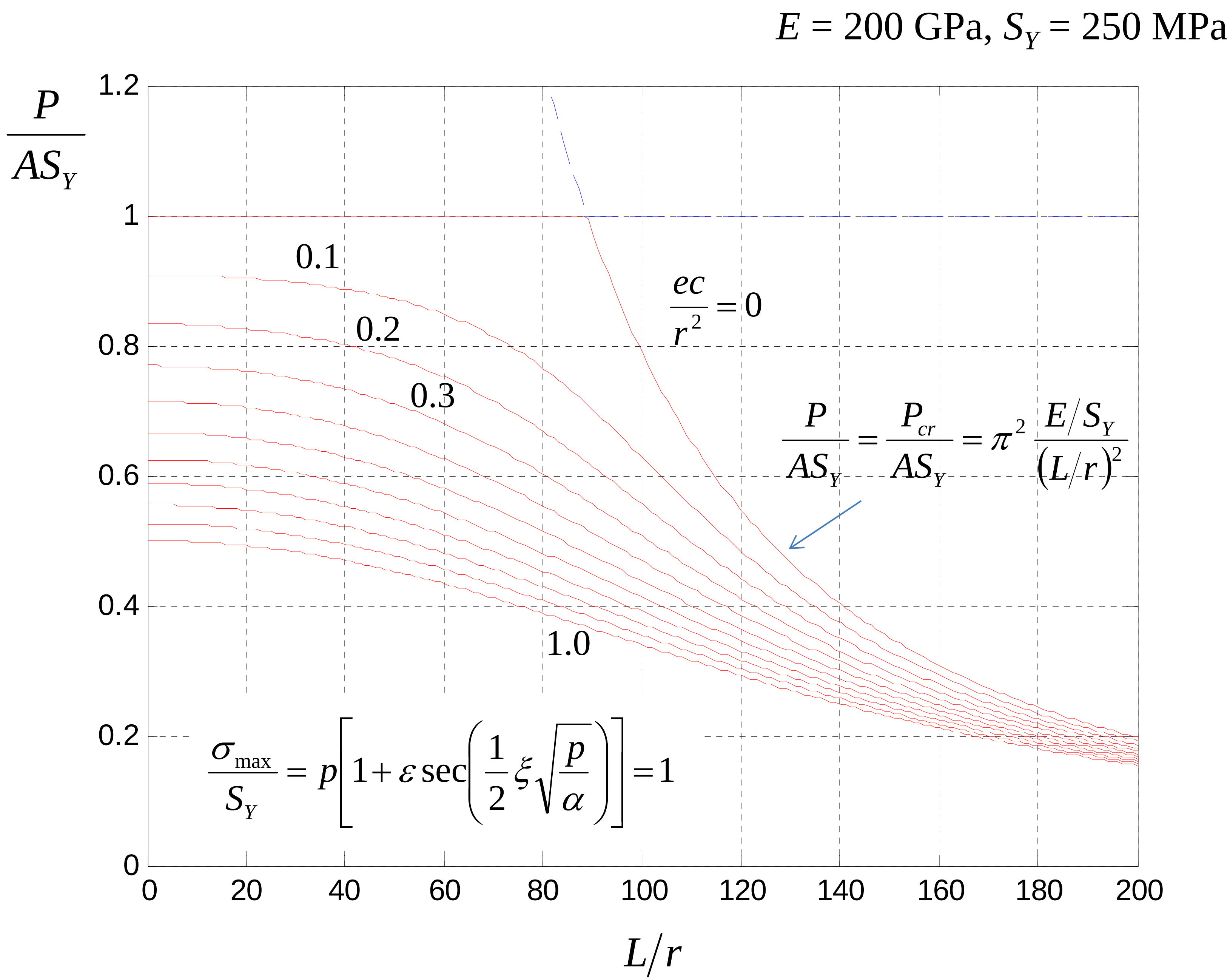
```matlab
clear;
e = 0:0.1:1;
x = 0.1:1:200.1;
E = 200e9;
sy = 250e6;
a = E/sy;
pi = acos(-1);
bcr = (pi^2)*a*(x.^(-2));

for i = 1:size(x,2)
    if bcr(i) >= 1
        p(i,1) = 1;
        indice = i;
    else
        p(i,1) = bcr(i);
    end
end
for j = 2:size(e,2)
    y0 = 1;
    for k = 1:size(x,2)
        y = fzero(@(y) buckling(y,x(k),e(j),a),y0);
        y0 = y;
        p(k,j) = y;
    end
end
figure(1), plot(x,p(:,:),'r'), axis([0 200 0 1.201]), hold;
figure(1), plot(x(indice:size(x,2)),ones(size(indice:size(x,2))),'--b');
figure(1), plot(x(indice-n:indice), bcr(indice-n:indice),'--b'); hold;
```

MATLAB®

Resolver:

$$p\left[1+\varepsilon\sec\left(\frac{1}{2}\xi\sqrt{\frac{p}{\alpha}}\right)\right]-1=0$$

```matlab
function f = buckling(p,x,e,a)
c = cos((x/2)*sqrt(p/a));
f = (p-1)*c + e*p;
```



$E = 200$ GPa, $S_Y = 250$ MPa
$\frac{P}{AS_Y}$
1.2
1
0.8
0.6
0.4
0.2
0
0 20 40 60 80 100 120 140 160 180 200
L/r
0.1
0.2
0.3
1.0
$\frac{ec}{r^2} = 0$
$\frac{P}{AS_Y} = \frac{P_{cr}}{AS_Y} = \pi^2 \frac{E/S_Y}{(L/r)^2}$
$\frac{\sigma_{\max}}{S_Y} = p\left[1 + \varepsilon \sec\left(\frac{1}{2}\xi\sqrt{\frac{p}{\alpha}}\right)\right] = 1$

$E = 200$ GPa, $S_Y = 250$ MPa

h =	2.5	mm
b =	25	mm
L =	30	mm
e =	0.17	mm
I =	32.6	mm^4
A =	62.5	mm^2
R =	0.72	mm
L/r =	41.6	
ec/r2 =	0.4