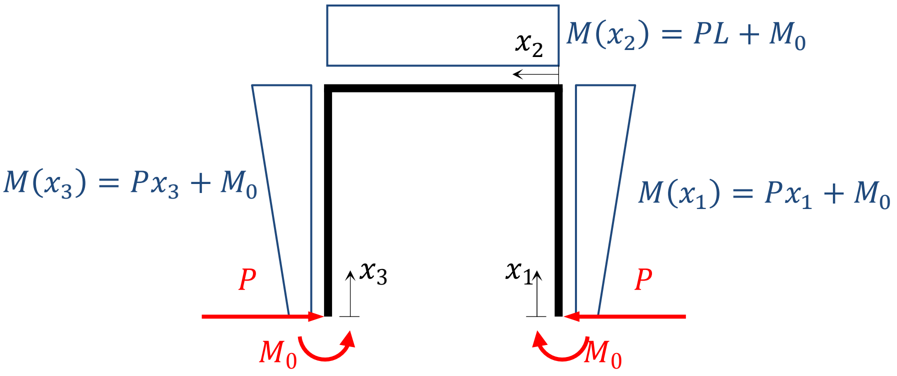
Problema 1. (3,0 pontos)

Distribuição do momento fletor



Teorema de Castigliano

$$\delta = \left.\frac{\partial U}{\partial P}\right|_{M_0=0} = \frac{1}{EI}\left[\int_0^L M\left.\frac{\partial M}{\partial P}\right|_{M_0=0} dx_1 + \int_0^L M\left.\frac{\partial M}{\partial P}\right|_{M_0=0} dx_2 + \int_0^L M\left.\frac{\partial M}{\partial P}\right|_{M_0=0} dx_3\right]$$

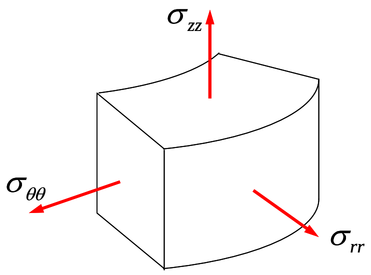
$$= \frac{1}{EI}\left[\int_0^L Px_1^2 dx_1 + \int_0^L PL^2 dx_2 + \int_0^L Px_3^2 dx_3\right] = \frac{1}{EI}\left[\frac{PL^3}{3} + PL^3 + \frac{PL^3}{3}\right] = \frac{5PL^3}{3EI}$$

$$\phi = \left.\frac{\partial U}{\partial M_0}\right|_{M_0=0} = \frac{1}{EI}\left[\int_0^L M\left.\frac{\partial M}{\partial M_0}\right|_{M_0=0} dx_1 + \int_0^L M\left.\frac{\partial M}{\partial M_0}\right|_{M_0=0} dx_2 + \int_0^L M\left.\frac{\partial M}{\partial M_0}\right|_{M_0=0} dx_3\right]$$

$$= \frac{1}{EI}\left[\int_0^L Px_1 dx_1 + \int_0^L PL dx_2 + \int_0^L Px_3 dx_3\right] = \frac{1}{EI}\left[\frac{PL^2}{2} + PL^2 + \frac{PL^2}{2}\right] = \frac{2PL^2}{EI}$$

Problema 2. (3,0 pontos)

Tensão	***Parede Interna do Tubo***
σ_1	$\sigma_1 = \sigma_{\theta\theta} = \frac{(b^2/a^2+1)}{(b^2/a^2-1)} P = 52{,}6\ MPa$
σ_2	$\sigma_2 = \sigma_{zz} = \frac{1}{(b^2/a^2-1)} P = 9{,}1\ MPa$
σ_3	$\sigma_3 = \sigma_{rr} = -P = -34{,}5\ MPa$
$\tau_{\max}$	$\tau_{\max} = \frac{1}{2}(\sigma_1 - \sigma_3) = 43{,}6\ MPa$
σ_{VM}	$\sigma_{VM} = 75{,}5\ MPa$

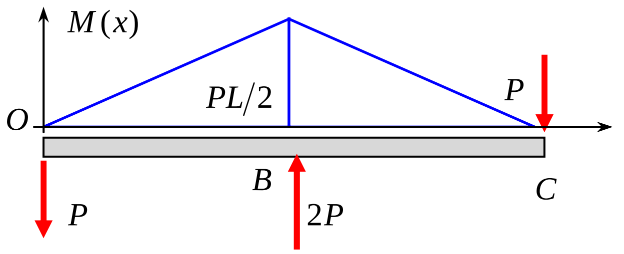


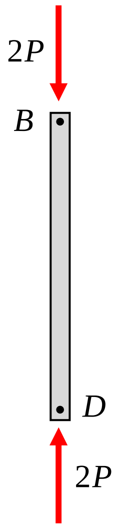
$$[\sigma] = \begin{bmatrix} \sigma_{rr} & 0 & 0 \\ 0 & \sigma_{\theta\theta} & 0 \\ 0 & 0 & \sigma_{zz} \end{bmatrix}$$

FS	Tresca	$F_{VM} = S_Y / \sigma_{VM} = 4{,}64$
	Von Mises	$F_T = S_Y / 2\tau_{max} = 4{,}02$

Problema 3. (2,0 pontos)

Equilíbrio e distribuição do momento fletor:





$$M(x) = \begin{cases} Px & 0 < x < L/2 \\ P(L-x) & L/2 < x < L \end{cases}$$

Mecanismos de colapso:

i. Colapso plástico na barra considerando seu material elástico/perfeitamente plástico

$$|\sigma| = \left|\frac{-2P}{A}\right| = Y_B \Rightarrow P_{L1} = \frac{Y_B A}{2} = \frac{Y_B a^2}{2} = 2{,}25\,\text{kN}$$

ii. Colapso plástico na viga considerando seu material elástico/perfeitamente plástico

$$M(L/2) = \frac{PL}{2} = M_L = \frac{3}{2} M_Y = \frac{Y_V a^3}{4} \Rightarrow P_{L2} = \frac{Y_V a^3}{2L} = 0{,}15\,\text{kN}$$

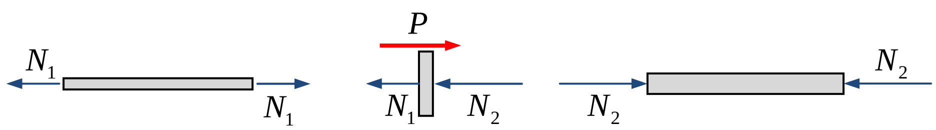
iii. Flambagem (instabilidade elástica) da barra (apoio simples-simples)

$$|N| = |-2P| = P_{cr} = \pi^2 \frac{EI}{(3L/2)^2} = \frac{\pi^2}{27}\frac{Ea^4}{L^2} \Rightarrow P_{L3} = \frac{\pi^2}{54}\frac{Ea^4}{L^2} = 0{,}57\,\text{kN}$$

$$\boxed{P_L = \min\{P_{L1}, P_{L2}, P_{L3}\} = P_{L2} = 0{,}15\,\text{kN}}$$

Problema 4. (2,0 pontos)

Equilíbrio: $N_1 + N_2 = P$



(i) No regime elástico:

$$\delta = \frac{N_1 L}{EA} = \frac{N_2 L}{2EA} \quad \Rightarrow \quad N_2 = 2N_1$$

$$N_1 + N_2 = P \quad \Rightarrow \quad N_1 = P/3 \quad \text{e} \quad N_2 = 2P/3$$

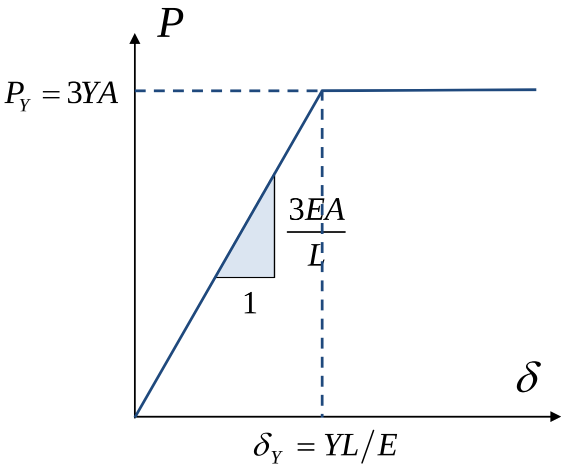
logo

$$\delta = \frac{L}{3EA} P$$

O escoamento das barras ocorrem simultaneamente quando $P = P_Y = 3YA$

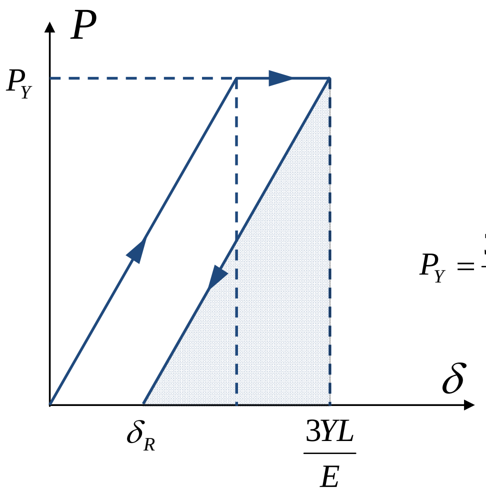
$$\sigma_1 = N_1/A = Y \quad \Rightarrow \quad P = P_Y = 3YA$$
$$\sigma_2 = -N_2/2A = -Y \quad \Rightarrow \quad P = P_Y = 3YA$$

O diagrama Força *vs.* Deslocamento é:



$$P(\delta) = \frac{3EA}{L}\delta, \quad \delta < \delta_Y = YL/E$$

Num ciclo de carregamento descarregamento com $\delta_{max} = 3YL/E$:



$$P_Y = \frac{3EA}{L}(3YL/E - \delta_R) \Rightarrow 3YA = \frac{3EA}{L}(3YL/E - \delta_R) \Rightarrow \delta_R = 2YL/E$$

O deslocamento residual é portanto $\delta_R = 2YL/E$.

As tensões residuais são nulas conforme mostram os gráficos tensão *vs.* deformação nas duas barras: