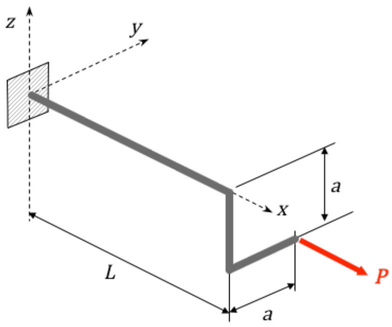


(i) Trecho AB

$$M_z(y) = -P(a-y)$$
$$\frac{\partial M_z}{\partial P} = -(a-y)$$
$$\frac{\partial U_{AB}}{\partial P} = \frac{1}{EI}\int_0^a P(a-y)^2 dy = \frac{Pa^3}{3EI}$$

(ii) Trecho BC

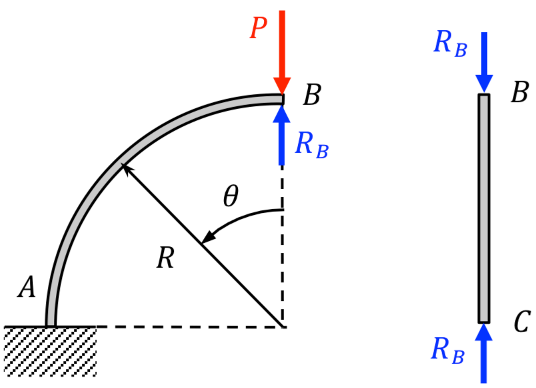
$$M_y(z) = -Pz \qquad M_z(z) = -Pa$$
$$\frac{\partial M_y}{\partial P} = -z \qquad \frac{\partial M_z}{\partial P} = -a$$
$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^a Pz^2 dz + \frac{1}{GJ}\int_0^a Pa^2 dz = \frac{Pa^3}{3EI} + \frac{Pa^3}{GJ}$$

(ii) Trecho CD

$$M_y(z) = -Pa \qquad M_z(z) = -Pa$$
$$\frac{\partial M_y}{\partial P} = -a \qquad \frac{\partial M_z}{\partial P} = -a$$
$$\frac{\partial U_{BC}}{\partial P} = \frac{1}{EI}\int_0^L Pa^2 dz + \frac{1}{EI}\int_0^L Pa^2 dz = \frac{2Pa^2L}{EI}$$

$$\delta_A = \frac{\partial U_{AB}}{\partial P} + \frac{\partial U_{BC}}{\partial P} + \frac{\partial U_{CD}}{\partial P} = \frac{Pa^3}{3EI} + \frac{Pa^3}{3EI} + \frac{Pa^3}{GJ} + \frac{2Pa^2L}{EI} = \frac{2Pa^3}{3EI}\left(1 + 3\frac{L}{a} + \frac{3}{2}\frac{EI}{GJ}\right)$$

Problema 2



Barra Circular ($V = P - R_B$):

$$\delta_B = \frac{\partial U}{\partial V} = \frac{1}{EI}\int_0^{\frac{\pi}{2}} M(\theta)\frac{\partial M(\theta)}{\partial V} R\, d\theta$$

$$M(\theta) = VR\sin\theta$$

Barra Vertical ($A = 100I/R^2$):

$$\delta_B = \frac{R}{EA}R_B = \frac{R^3}{100EI}R_B$$

$$\delta_B = \frac{VR^3}{EI}\int_0^{\frac{\pi}{2}} \sin^2\theta \, d\theta = \frac{\pi R^3}{4EI}V = \frac{\pi R^3}{4EI}(P - R_B)$$

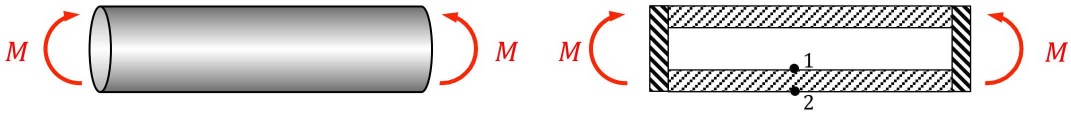
Portanto:

$$\frac{\pi R^3}{4EI}(P - R_B) = \frac{R^3}{100EI}R_B \quad \Rightarrow \quad R_B = \frac{25\pi}{1 + 25\pi}P = 0{,}987P$$

Flambagem:

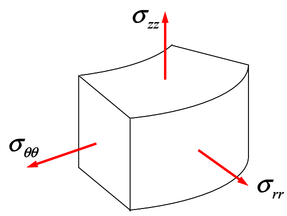
$$R_B = 0{,}987P < \pi^2 \frac{EI}{R^2} \quad \Rightarrow \quad R_B < 10{,}0\frac{EI}{R^2}$$

Problema 3



(a)

Parede Interna ($r = a$)	Parede Externa ($r = b$)
$\sigma_{rr}(a) = -P = -60{,}0 \text{ MPa}$	$\sigma_{rr}(b) = 0$
$\sigma_{\theta\theta}(a) = \dfrac{b^2/a^2 + 1}{b^2/a^2 - 1}P = 100 \text{ MPa}$	$\sigma_{\theta\theta}(b) = \dfrac{2}{b^2/a^2 - 1}P = 40{,}0 \text{ MPa}$
$\sigma_{zz}(a) = \dfrac{P}{b^2/a^2 - 1} + \dfrac{aM}{\pi(b^4 - a^4)/4} = 87{,}9 \text{ MPa}$	$\sigma_{zz}(b) = \dfrac{P}{b^2/a^2 - 1} + \dfrac{bM}{\pi(b^4 - a^4)/4} = 156 \text{ MPa}$



(b)

Parede Interna ($r = a$)	Parede Externa ($r = b$)
$\sigma_1 = \sigma_{\theta\theta}(a) = 100 \text{ MPa}$	$\sigma_1 = \sigma_{zz}(b) = 156 \text{ MPa}$
$\sigma_2 = \sigma_{zz}(a) = 87{,}9 \text{ MPa}$	$\sigma_2 = \sigma_{\theta\theta}(b) = 40{,}0 \text{ MPa}$
$\sigma_3 = \sigma_{rr}(a) = -60{,}0 \text{ MPa}$	$\sigma_3 = \sigma_{rr}(b) = 0$
$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 80{,}0 \text{ MPa}$	$\tau_{max} = \dfrac{\sigma_1 - \sigma_3}{2} = 78 \text{ Mpa}$
$\sigma_{VM} = \sqrt{\dfrac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]} = 154 \text{ MPa}$	$\sigma_{VM} = \sqrt{\dfrac{1}{2}[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2]} = 140 \text{ Mpa}$
$n_{Tresca} = \dfrac{S_Y/2}{\max\{\tau_{max}\}} = 4{,}00$	
$n_{VM} = \dfrac{S_Y}{\max\{\sigma_{VM}\}} = 4{,}15$	