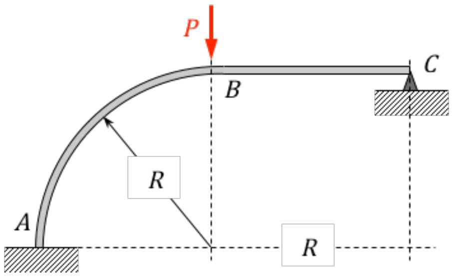
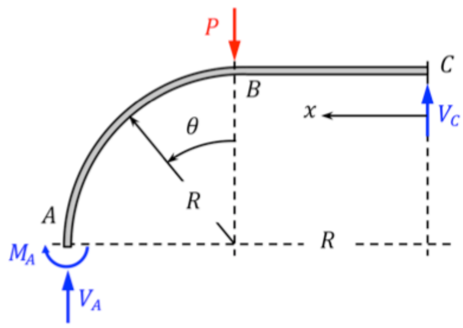
Problema 1 (3,5 pontos).





(a) Reações em A e C

$$M(s) = \begin{cases} V_C x, \ \ 0 \le x \le R \text{ (trecho } CB) \\ V_C R(1+\sin\theta) - PR\sin\theta, \ \ 0 \le \theta \le \pi/2 \ \text{ (trecho } BA) \end{cases}$$

$$\frac{\partial M}{\partial V_C} = \begin{cases} x, \ \ 0 \le x \le R \text{ (trecho } CB) \\ R(1+\sin\theta), \ \ 0 \le \theta \le \pi/2 \ \text{ (trecho } BA) \end{cases}$$

$$\frac{\partial M}{\partial P} = \begin{cases} 0, \ \ 0 \le x \le R \text{ (trecho } CB) \\ -R\sin\theta, \ \ 0 \le \theta \le \pi/2 \ \text{ (trecho } BA) \end{cases}$$

$$\delta_C = \frac{\partial U}{\partial V_C} = \frac{1}{EI}\left[V_C\left(\int_0^R x^2 dx\right) + V_C R^3\left(\int_0^{\pi/2}(1+\sin\theta)^2 d\theta\right) - PR^3\left(\int_0^{\pi/2}(1+\sin\theta)\sin\theta\, d\theta\right)\right]$$

$$\int_0^{\pi/2}(1+\sin\theta)\sin\theta\, d\theta = 1 + \frac{\pi}{4}$$

$$= \frac{1}{EI}\left[\frac{V_C R^3}{3} + \frac{V_C R^3(3\pi+8)}{4} - \frac{PR^3(\pi+4)}{4}\right] = 0 \ \ \Rightarrow \ \ V_C = \frac{3(\pi+4)}{9\pi+28}P$$

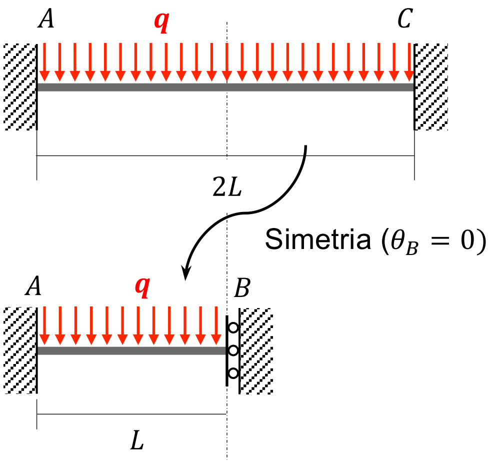
$$V_A = \frac{6\pi+16}{9\pi+28}P \ \ e \ \ M_A = (2V_A - P)R = \frac{3\pi+4}{9\pi+28}PR$$

(b) Deslocamento em B:

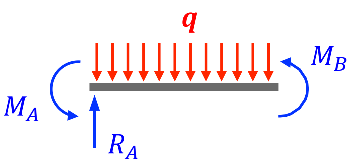
$$\delta_B = \frac{\partial U}{\partial P} = \frac{PR^3}{EI}\left[-\frac{3(\pi+4)}{9\pi+28}\int_0^{\pi/2}(1+\sin\theta)\sin\theta\, d\theta + \int_0^{\pi/2}\sin^2\theta\, d\theta\right]$$

$$= \frac{PR^3}{EI}\left[\frac{\pi}{4} - \frac{3(\pi+4)^2}{4(9\pi+28)}\right] = 0{,}106\frac{PR^3}{EI}$$

Problema 2 (3,5 pontos).



Equilíbrio



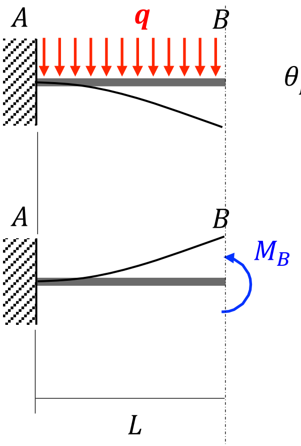
$$R_A = qL$$

$$M_A + M_B = \frac{qL^2}{2}$$

(a) Valor de q_Y

No regime elástico:

Rotações: $\theta_B^{(1)} = \theta_B^{(2)}$



$$\theta_B^{(1)} = \frac{qL^3}{6EI}$$

$$\theta_B^{(2)} = \frac{M_B L}{EI}$$

$$\theta_B = 0 \;\Rightarrow\; \frac{qL^3}{6EI} = \frac{M_B L}{EI} \;\Rightarrow\; M_B = \frac{qL^2}{6}$$

Logo

$$M_A = \frac{qL^2}{2} - M_B = \frac{qL^2}{3}$$

e

$$\max\{|M(x)|\} = |M(0)| = \frac{qL^2}{3}$$

No início do escoamento $q = q_Y$ e $\max\{|M(x)|\} = M_Y$, portanto:

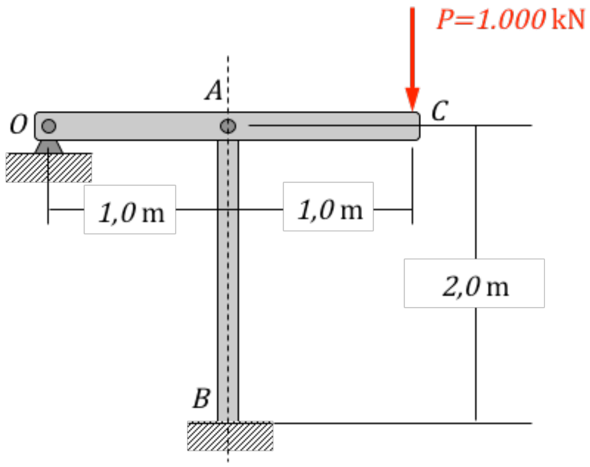
$$\frac{q_Y L^2}{3} = M_Y = \frac{S_Y b h^2}{6} \;\Rightarrow\; q_Y = \frac{S_Y b h^2}{2L^2}$$

(b) Valor de q_L:

O colapso plástico ocorre quando rótulas plásticas se formam em A e B, ou seja, $M_A = M_B = M_L$, portanto:

$$M_A + M_B = 2M_L = \frac{q_L L^2}{2} \quad \Rightarrow \quad q_L = \frac{4M_L}{L^2} = \frac{S_Y b h^2}{L^2}$$

Problema 3 (3,0 pontos).



O esforço axial compressivo atuando na barra AB é $N_{AB} = -2P$. Portanto, para que não ocorra flambagem, e utilizando um coeficiente de segurança $n = 4{,}0$, devemos ter (onde $L = 2$ m):

$$|N_{AB}| = 2P < \frac{P_{cr}}{n} = \frac{20{,}2}{n}\frac{EI}{L^2} \quad \Rightarrow \quad I = \frac{\pi}{64}(D_E^4 - D_i^4) > \frac{2nPL^2}{20{,}2E} = 2{,}20\times10^{-5}\ \mathrm{m}^4$$

$$D_i^4 < D_e^4 - 2{,}20\times10^{-5}\frac{64}{\pi} \quad \Rightarrow \quad D_i = D_e - 2t < 184\ \mathrm{mm} \quad \Rightarrow \quad t > 7{,}9\ \mathrm{mm}$$