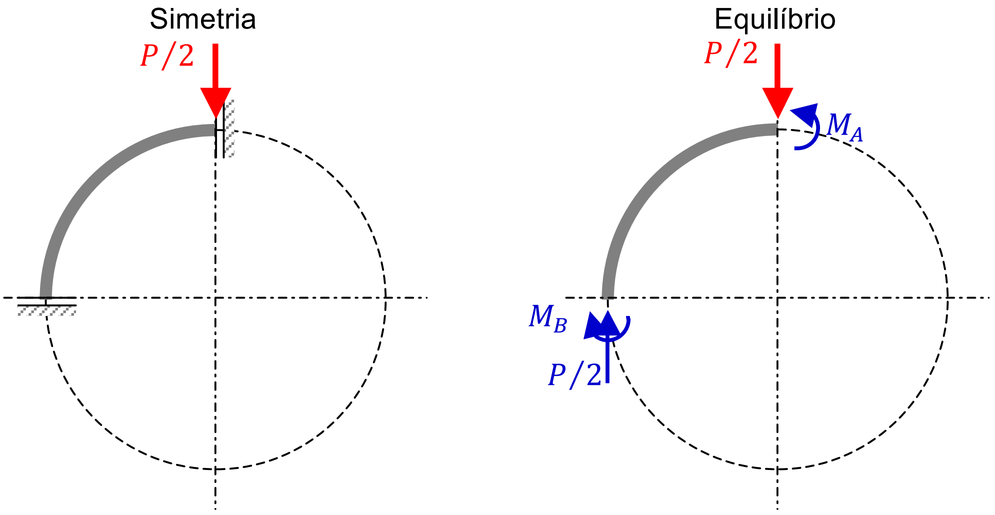
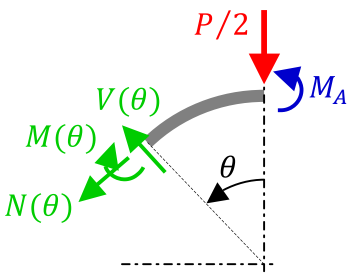
MECÂNICA DOS SÓLIDOS II

Gabarito P2 – 21/11/2011

Problema 1. (3,5 pontos)



Distribuição do Momento Fletor



$$M(\theta) = M_A - \frac{P}{2} R \sin\theta\,, \quad 0 < \theta < \pi/2$$

$$\frac{\partial M}{\partial M_A} = 1$$

$$\phi_A = \frac{\partial U}{\partial M_A} = 0$$

logo

$$\frac{\partial U}{\partial M_A} = \frac{1}{EI}\int_0^{\pi/2} M(\theta)\frac{\partial M}{\partial M_A} R d\theta = \frac{1}{EI}\int_0^{\pi/2}\left[M_A - \frac{P}{2}R\sin\theta\right] R d\theta = \frac{R}{EI}\left[\frac{M_A\pi}{2} - \frac{PR}{2}\right] = 0 \;\Rightarrow\; M_A = \frac{PR}{\pi}$$

$$M_B = M(\pi/2) = M_A - PR/2 = -\frac{(\pi-2)}{2}\frac{PR}{\pi}$$

Distribuição de tensões

$$\sigma_{\theta\theta}(y,\theta) = -y\frac{M(\theta)}{I} + \frac{N(\theta)}{A}, \quad -a/2 < y < a/2 \quad (I = a^4/12\,,\; A = a^2)$$

Na Seção A $(\theta = 0)$

$$\sigma_{\theta\theta}(y,0) = -y\frac{M_A}{I} = -12y\frac{PR}{\pi a^4}$$

$$\max\{|\sigma_{\theta\theta}(y,0)|\} = \left|\sigma_{\theta\theta}\left(\pm\frac{a}{2},0\right)\right| = \frac{6PR}{\pi a^3}$$

Na Seção B ($\theta = \pi/2$)

$$\sigma_{\theta\theta}\left(y, \frac{\pi}{2}\right) = -y\frac{M_B}{I} - \frac{P/2}{A} = 6(\pi-2)y\frac{PR}{\pi a^4} - \frac{P}{2a^2}$$

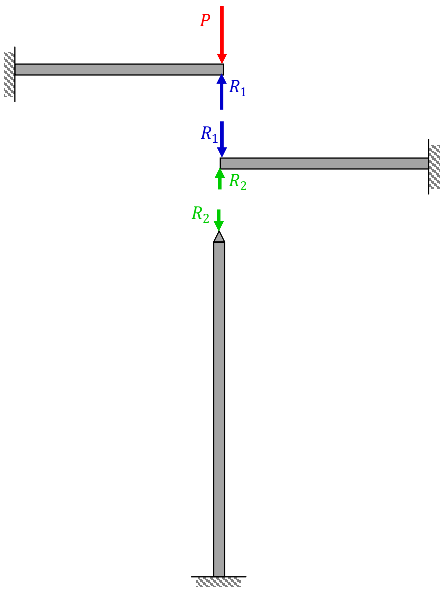
$$max\{|\sigma_{\theta\theta}(y, \pi/2)|\} = \left|\sigma_{\theta\theta}\left(-\frac{a}{2}, 0\right)\right| = \left[\frac{(\pi-2)}{2} + \frac{\pi}{12}\frac{a}{R}\right]\frac{6PR}{\pi a^3}$$

Para vigas curvas esbeltas $a/R \ll 1$ e, portanto, $[(\pi-2)/2 + \pi(a/R)/12] < 1$. Nesse caso, a máxima tensão de flexão, em valor absoluto, ocorre na Seção A.

Problema 2 (3,0 pontos).

Compatibilidade de deformações

$$\frac{(\mathrm{P}-\mathrm{R}_1)\mathrm{L}^3}{3\mathrm{EI}} = \frac{(\mathrm{R}_1-\mathrm{R}_2)\mathrm{L}^3}{3\mathrm{EI}} = \frac{\mathrm{R}_2(3\mathrm{L})}{\mathrm{EA}}$$



Logo

$$P - R_1 = R_1 - R_2 = 9\left(\frac{I/A}{L^2}\right)R_2$$

$$R_2 = \frac{P}{1+18\xi}, \quad \text{onde } \xi = \frac{I/A}{L^2}$$

$$R_2 < \frac{\pi^2}{4}\frac{EI}{(3L)^2} \Rightarrow P < (1+18\xi)\frac{\pi^2}{4}\frac{EI}{9L^2} = 8{,}2 \text{ kN}$$

$P < 8{,}2$ kN

Problema 3 (3,5 pontos)

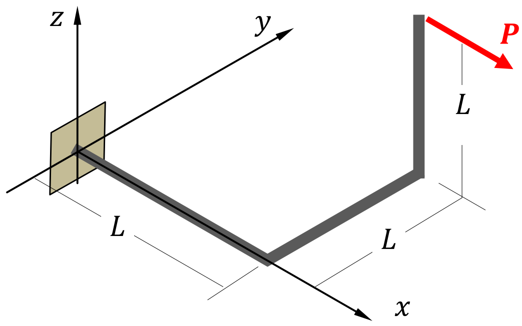
$$\delta_A = \frac{\partial U}{\partial P} = \frac{1}{EI}\int M\frac{\partial M}{\partial P}ds + \frac{1}{GJ}\int T\frac{\partial T}{\partial P}ds$$

(a) Trecho paralelo à direção z ($0 < z < L$)

$M_y(z) = -Pz \qquad \partial M_y/\partial P = -z$

$M_x(z) = 0$

$T(z) = M_z(z) = 0$



(b) Trecho paralelo à direção y ($0 < y < L$)

$M_z(y) = -Py \qquad \partial M_z/\partial P = -y$

$M_x(y) = 0$

$T(y) = M_z(y) = PL \qquad \partial T/\partial P = L$

(c) Trecho paralelo à direção x $(0 < x < L)$

$M_z(x) = -PL \qquad \partial M_z/\partial P = -L$

$M_y(x) = PL \qquad \partial M_y/\partial P = L$

$T(x) = M_x(x) = 0$

$$\delta_A = \frac{1}{EI}\left[\int_0^L (-Pz)(-z)dz + \int_0^L (-Py)(-y)dy + \int_0^L (-PL)(-L)dx + \int_0^L (PL)(L)dx\right] + \frac{1}{GJ}\int_0^L (PL)(L)dy$$

$$= \frac{1}{EI}\left[\frac{PL^3}{3} + \frac{PL^3}{3} + PL^3 + PL^3\right] + \frac{1}{GJ}PL^3$$

$$\delta_A = \frac{8PL^3}{3EI} + \frac{PL^3}{GJ} = \frac{64PL^3}{3\pi ED^4}(11 + 3\nu)$$