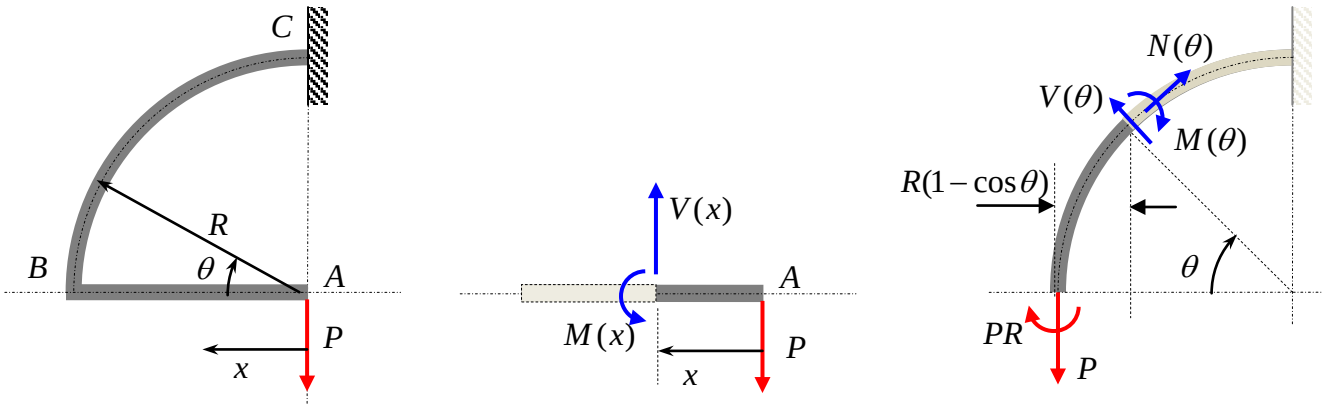
Problema 1. (3,5 pontos)

Distribuição do Momento Fletor



$$M(x) = Px, \quad 0 < x < R \qquad M(\theta) = -PR\cos\theta, \quad 0 < \theta < \pi/2$$

Aplicação do Teorema de Castigliano

Desprezando-se a contribuição dos esforços normal e cortante, a energia de deformação é dada por

$$U(P) = \int \frac{M^2(s,P)}{2EI} ds$$

Deslocamento:

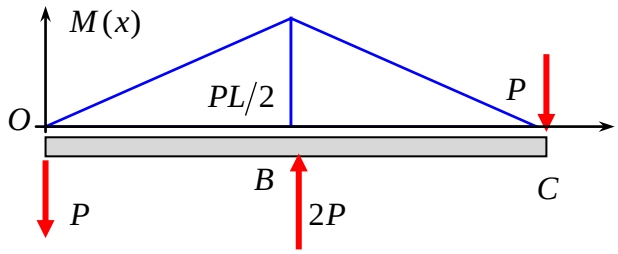
$$\delta = \frac{\partial U}{\partial P} = \frac{1}{EI}\int M \frac{\partial M}{\partial P} ds$$

Logo:

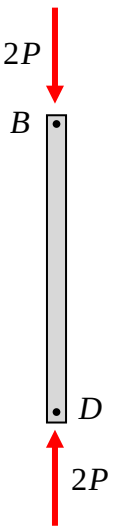
$$\delta = \frac{1}{EI}\left[\int_0^R (Px)(x)dx + \int_0^{\pi/2} (-PR\cos\theta)(-R\cos\theta)Rd\theta\right] = \frac{P}{EI}\left(\frac{x^3}{3}\right)_0^R + \frac{PR^3}{EI}\left(\frac{\theta}{2} + \frac{\sin 2\theta}{4}\right)_0^{\pi/2} = \frac{PR^3}{3EI}\left(1 + \frac{3\pi}{4}\right)$$

Problema 2. (4,0 pontos)

(a) Equilíbrio e distribuição de momento fletor:

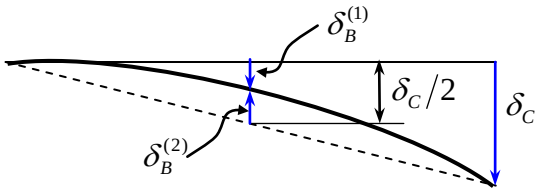


$$M(x) = \begin{cases} Px & 0 < x < L/2 \\ P(L-x) & L/2 < x < L \end{cases}$$



(b) Deflexão vertical no ponto C

Pelo método direto (superposição):



$$\delta_B^{(1)} = \frac{(2P)(3L/2)}{EA}$$ Barra carregada axialmente

$$\delta_B^{(2)} = \frac{(2P)L^3}{48EI}$$ Viga simplesmente apoiada

$$\frac{\delta_C}{2} = \delta_B^{(1)} + \delta_B^{(1)} = \frac{3PL}{EA} + \frac{PL^3}{24EI} \Rightarrow \delta_C = \frac{6PL}{EA} + \frac{PL^3}{12EI} = \frac{PL^3}{a^4E}\left(1 + 6\frac{a^2}{L^2}\right)$$

ou através do teorema de Castigliano:

$$U(P) = \int_0^L \frac{M^2(x)}{2EI}dx + \int_0^{3L/2} \frac{N^2(x)}{2EA}dx + = 2\int_0^{L/2} \frac{(Px)^2}{2EI}dx + \int_0^{3L/2} \frac{(-2P)^2}{2EA}dx = \frac{P^2L^3}{24EI} + \frac{3P^2L}{EA}$$

$$\delta_C = \frac{\partial U}{\partial P} = \frac{PL^3}{12EI} + \frac{6PL}{EA} = \frac{PL^3}{a^4E}\left(1 + 6\frac{a^2}{L^2}\right)$$

(c) Mecanismos de colapso:

i. Colapso plástico na barra considerando seu material elástico/perfeitamente plástico

$$|\sigma| = \left|\frac{-2P}{A}\right| = Y_B \Rightarrow P_{L1} = \frac{Y_B A}{2} = \frac{Y_B a^2}{2}$$

ii. Colapso plástico na viga considerando seu material elástico/perfeitamente plástico

$$M(L/2) = \frac{PL}{2} = M_L = \frac{3}{2}M_Y = \frac{Y_V a^3}{4} \Rightarrow P_{L2} = \frac{Y_V a^3}{2L}$$

iii. Flambagem (instabilidade elástica) da barra (apoio simples-simples)

$$|N| = |-2P| = P_{cr} = \pi^2 \frac{EI}{(3L/2)^2} = \frac{\pi^2}{27}\frac{Ea^4}{L^2} \Rightarrow P_{L3} = \frac{\pi^2}{54}\frac{Ea^4}{L^2}$$

Problema 3.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} + \alpha\,\Delta T = 0 \Rightarrow \frac{N/A}{E} = -\alpha\,\Delta T \Rightarrow N = -\alpha EA\Delta T$$

$$|N| < P_{cr} = 4\pi^2\frac{EI}{L^2} \quad \Rightarrow \quad \alpha EA\Delta T < 4\pi^2\frac{EI}{L^2} \quad \Rightarrow \quad L < 2\pi\sqrt{\frac{I}{\alpha A\Delta T}} = \frac{\pi D}{\sqrt{2\alpha\,\Delta T}} = 1{,}81\,\text{m}$$