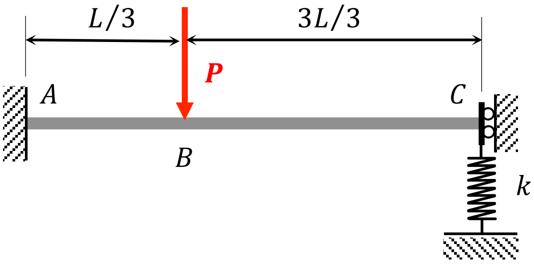
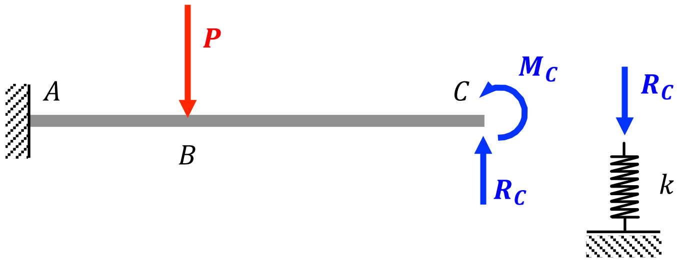
MECÂNICA DOS SÓLIDOS II
P1 – 06/10/2017 – Gabarito

Problema 1.





(a) Para $k = 3EI/5L^3$, determine a carga P_Y que leva ao início do escoamento na estrutura. (3,0 pontos).

$$\delta_C = \delta_C^{(1)} - \delta_C^{(2)} - \delta_C^{(3)} = \frac{R_C}{k} = \frac{5R_CL^3}{3EI} \qquad \phi_C = \phi_C^{(1)} - \phi_C^{(2)} - \phi_C^{(3)} = 0$$

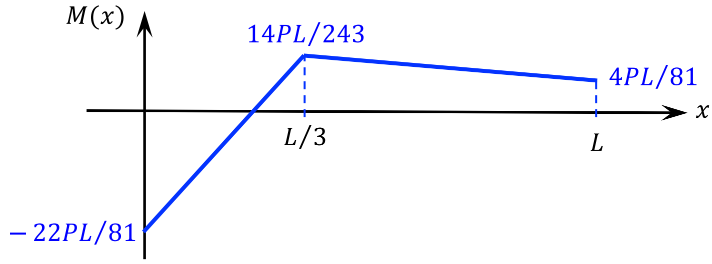
$$\delta_C^{(1)} = \frac{P(L/3)^2(3L - L/3)}{6EI} = \frac{4PL^3}{81EI} \qquad \phi_C^{(1)} = \frac{P(L/3)^2}{2EI} = \frac{PL^3}{18EI}$$

$$\delta_C^{(2)} = \frac{R_CL^3}{3EI} \qquad \phi_C^{(2)} = \frac{R_CL^2}{2EI}$$

$$\delta_C^{(3)} = \frac{M_CL^2}{2EI} \qquad \phi_C^{(3)} = \frac{M_CL}{EI}$$

$$\begin{cases} \frac{4PL^3}{81EI} - \frac{R_CL^3}{3EI} - \frac{M_CL^2}{2EI} = \frac{5R_CL^3}{3EI} \\ \frac{PL^3}{18EI} - \frac{R_CL^2}{2EI} - \frac{M_CL}{EI} = 0 \end{cases} \Rightarrow \begin{cases} 2R_CL + \frac{M_C}{2} = \frac{4PL}{81} \\ \frac{R_CL}{2} + M_C = \frac{PL}{18} \end{cases} \Rightarrow \begin{cases} R_C = \frac{P}{81} \\ M_C = \frac{4PL}{81} \end{cases}$$

Diagrama de momento fletor:

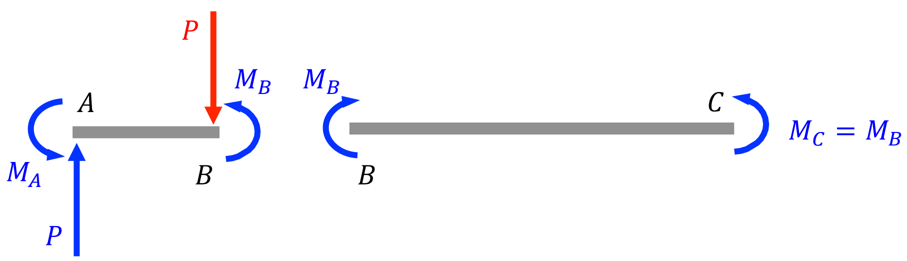


$$P = P_Y \Rightarrow \max\{|M(x)|\} = M_Y$$

Logo

$$\frac{22P_YL}{81} = \frac{Ya^3}{6} \Rightarrow P_Y = \frac{27}{44}\frac{Ya^3}{L}$$

(b) Considere agora $k \to 0$. Determine a carga de colapso plástico P_L (2,0 pontos).



O Colapso plástico ocorre quando $M_A = M_B = M_L$. Portanto, como $PL/3 = M_A + M_B$, obtém-se:

$$P_L = 6M_L/L = 3Ya^3/2L$$

Problema 2. (2,0 pontos)

$$\frac{d^2v}{dx^2} = \kappa \quad \Rightarrow \quad v(x) = \frac{\kappa x^2}{2} + ax + b$$

$$v(0) = 0 \quad \Rightarrow \quad b = 0$$

$$v(L) = 0 \quad \Rightarrow \quad a = -\frac{\kappa L}{2}$$

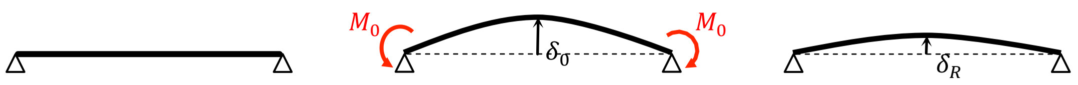
Portanto

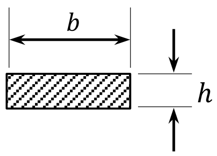
$$v(x) = -\frac{\kappa L^2}{2}\left(\frac{x}{L}\right)\left(1 - \frac{x}{L}\right)$$

Derivando-se a expressão acima, observa-se que $dv/dx = 0$ em $x = L/2$, logo

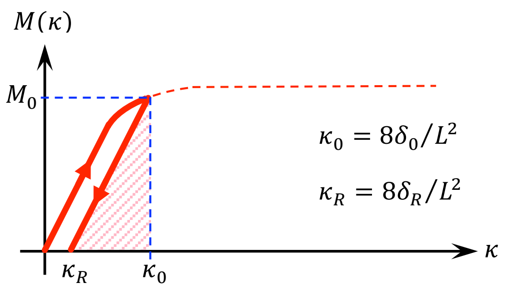
$$\max\{|v(x)|\} = |v(L/2)| = \frac{\kappa L^2}{8}$$

Problema 3. (3,0 pontos)





δ_R (mm)	L (mm)	b (mm	h (mm)	E (GPa)	S_Y (Mpa)
50	450	12	7,0	200	1200



$$M(\kappa) = \begin{cases} EI\kappa\,, & \kappa < \kappa_Y \\ \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa}\right)^2\right], & \kappa > \kappa_Y \end{cases}$$

$\kappa_Y = 2S_Y/Eh$ e $M_Y = 2S_Y I/h$

$$EI(\kappa_0 - \kappa_R) = M_0 = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa_0}\right)^2\right]$$

Logo

$$\kappa_0 = \kappa_R + \frac{3M_Y}{2EI}\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa_0}\right)^2\right]$$

A equação acima pode ser resolvida de forma iterativa:

$$\kappa_0^{(1)} = \kappa_R + \frac{3M_Y}{2EI}$$

Enquanto $\left|\kappa_0^{(n+1)} - \kappa_0^{(n)}\right| < \text{tol}$, repetir:

$$\kappa_0^{(n+1)} = \kappa_R + \frac{3M_Y}{2EI}\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa_0^{(n)}}\right)^2\right]$$

Para os dados do problema:

$$I = \frac{bh^3}{12} = 3{,}43\times10^{-10}\ \text{m}^4$$

$$\kappa_Y = \frac{2S_Y}{Eh} = 1{,}71\ \text{m}^{-1}$$

$$M_Y = \frac{2S_Y I}{h} = 118\ \text{N·m}$$

$$\kappa_R = \frac{8\delta_0}{L^2} = 1{,}98\ \text{m}^{-1}$$

Portanto, considerando uma tolerância de $5{,}00\times10^{-2}\ \text{m}^{-1}$ (três algarismos significativos):

$$\kappa_0^{(1)} = \kappa_R + \frac{3M_Y}{2EI} = 4{,}55\ \text{m}^{-1}$$

$$\kappa_0^{(2)} = \kappa_R + \frac{3M_Y}{2EI}\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa_0^{(1)}}\right)^2\right] = 4{,}42\ \text{m}^{-1} \quad \text{e} \quad \left|\kappa_0^{(2)} - \kappa_0^{(1)}\right| = 1{,}22\times10^{-1}\ \text{m}^{-1}$$

$$\kappa_0^{(3)} = \kappa_R + \frac{3M_Y}{2EI}\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa_0^{(2)}}\right)^2\right] = 4{,}42\ \text{m}^{-1} \quad \text{e} \quad \left|\kappa_0^{(3)} - \kappa_0^{(2)}\right| = 6{,}80\times10^{-3}\ \text{m}^{-1}$$

Já na primeira iteração a solução converge para $\kappa_0 = 4{,}42\ \text{m}^{-1}$, correspondendo a $\delta_0 = \kappa_0 L^2/8 = 112\ \text{mm}$.

O valor de M_0 é calculado abaixo:

$$M_0 = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{\kappa_Y}{\kappa_0}\right)^2\right] = 168\ \text{N·m}$$

A distribuição da tensão residual é dada pela equação:

$$\sigma_{xx}^R(y) = \begin{cases} -S_Y - yM_0/I\,, & -h/2 < y < -r_Y \\ y\,S_Y/r_Y - yM_0/I\,, & -r_Y < y < r_Y \\ S_Y - yM_0/I\,, & r_Y < y < h/2 \end{cases}$$

onde $r_Y = h\kappa_Y/2\kappa_0 = 1{,}36\ \text{mm}$.

Portanto, com y em mm e σ_{xx}^R em MPa:

$$\sigma_{xx}^R(y) = \begin{cases} -1200 - 489y\,, & -3{,}5 < y < -1{,}36\ \text{mm} \\ 396y\,, & -1{,}36 < y < 1{,}36\ \text{mm} \\ 1200 - 489y & 1{,}36 < y < 3{,}5\ \text{mm} \end{cases}$$