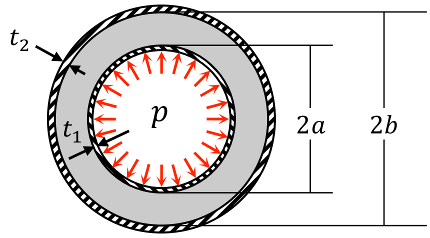
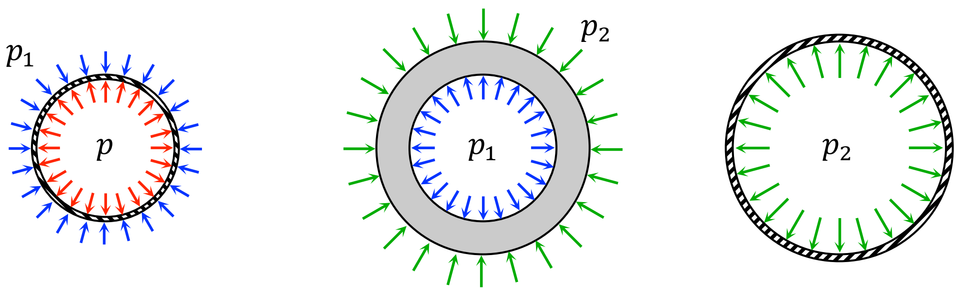
MECÂNICA DOS SÓLIDOS II

Gabarito P1 – 25/09/2015

Problema 1 (3,5 pontos)





a	b	t_1	t_2	E_{Al}	ν_{Al}	E_{PTFE}	ν_{PTFE}
30 mm	45 mm	2 mm	3 mm	70 GPa	0,30	0,4 GPa	0,46

Tubo 1 (parede fina – $a \gg t_1$):

$$\frac{\Delta a}{a} = \frac{(p - p_1)a}{E_{aço} t_1} = C_1(p - p_1)$$

Tubo 2 (parede grossa):

$$\frac{\Delta a}{a} = \frac{(1 - \nu_{ptfe}) + (1 + \nu_{ptfe})(b^2/a^2)}{E_{ptfe}(b^2/a^2 - 1)} p_1 - \frac{2(b^2/a^2)}{E_{ptfe}(b^2/a^2 - 1)} p_2 = A_1 p_1 - B_1 p_2$$

$$\frac{\Delta b}{b} = \frac{2}{E_{ptfe}(b^2/a^2 - 1)} p_1 - \frac{(1 - \nu_{ptfe})(b^2/a^2) + (1 + \nu_{ptfe})}{E_{ptfe}(b^2/a^2 - 1)} p_2 = A_2 p_1 - B_2 p_2$$

Tubo 3 (parede fina – $b \gg t_2$):

$$\frac{\Delta b}{b} = \frac{p_2 b}{E_{aço} t_2} = C_2 p_2$$

Onde:

$$C_1 = \frac{a/t_1}{E_{aço}} = 2{,}1\times10^{-10}\ \text{MPa}^{-1}$$

$$A_1 = \frac{(1 - \nu_{ptfe}) + (1 + \nu_{ptfe})(b^2/a^2)}{E_{ptfe}(b^2/a^2 - 1)} = 7{,}7\times10^{-9}\ \text{MPa}^{-1}$$

$$B_1 = \frac{2(b^2/a^2)}{E_{ptfe}(b^2/a^2 - 1)} = 9{,}0\times10^{-9}\ \text{MPa}^{-1}$$

$$A_2 = \frac{2}{E_{ptfe}(b^2/a^2 - 1)} = 4{,}0\times10^{-9}\ \text{MPa}^{-1}$$

$$B_2 = \frac{\left(1 - \nu_{ptfe}\right)(b^2/a^2) + \left(1 + \nu_{ptfe}\right)}{E_{ptfe}(b^2/a^2 - 1)} = 5{,}4\times10^{-9}\ \text{MPa}^{-1}$$

$$C_2 = \frac{b/t_2}{E_{aço}} = 2{,}1\times10^{-10}\ \text{MPa}^{-1}$$

Portanto:

$$A_1 p_1 - B_1 p_2 = C_1(p - p_1)$$

$$A_2 p_1 - B_2 p_2 = C_2 p_2$$

logo

$$p_2 = \frac{A_2}{B_2 + C_2} p_1$$

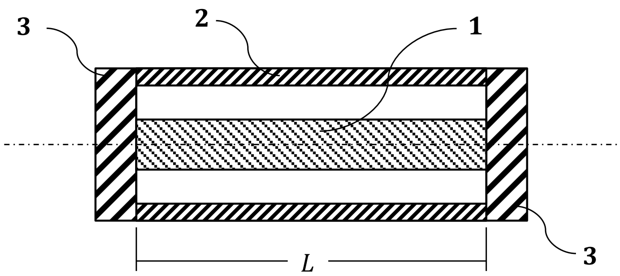
$$p_1 = \frac{C_1}{(A_1 + C_1) - \left(\frac{B_1 A_2}{B_2 + C_2}\right)} p$$

Substituindo os valores obtém-se:

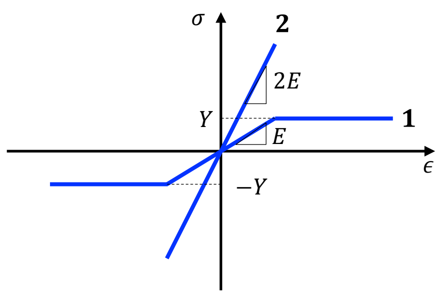
$$p_1 = 0{,}15\,p = \ 1{,}1\ \text{MPa}$$

$$p_2 = 0{,}11\,p = \ 0{,}77\ \text{MPa}$$

Problema 2 (3,0 pontos)



$$A_1 = A_2 = A$$
$$E_1 = E$$
$$E_2 = 2E$$
$$\alpha_1 = \alpha$$
$$\alpha_2 = 2\alpha$$



Para $\Delta T < \Delta T_y$ (o eixo **1** e o tubo **2** estão no regime elástico)

$$\left.\begin{aligned} & N_1 + N_2 = 0 \\ & \frac{\delta_1}{L} = \frac{N_1}{EA} + \alpha\Delta T \\ & \frac{\delta_2}{L} = \frac{N_2}{2EA} + 2\alpha\Delta T \\ & \delta_1 = \delta_2 = \delta \end{aligned}\right\} \Rightarrow \quad \begin{aligned} & N_1 = \frac{2\alpha EA}{3}\Delta T \\ & \delta = \frac{5\alpha L}{3}\Delta T \end{aligned}$$

O escoamento tem início na Barra 1 quando $N_1 = YA$, logo

(a) $\quad \Delta T_y = \dfrac{3Y}{2\alpha E}$

(b) $\quad \delta_y = \dfrac{5YL}{2E}$

Quando $\Delta T > \Delta T_y$ o eixo **1** passa a trabalhar no regime plástico, portanto

$$N_1 = YA \Rightarrow N_2 = -N_1 = -YA$$

$$\delta = \delta_2 = \frac{N_2 L}{2EA} + 2\alpha L \Delta T = -\frac{YL}{2E} + 2\alpha L \Delta T$$

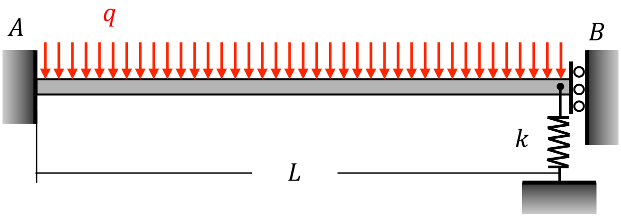
Logo

$$\delta(\Delta T) = \begin{cases} \frac{5\alpha L}{3}\Delta T, & \Delta T \leq \Delta T_y = 3Y/2\alpha E \\ -\frac{YL}{2E} + 2\alpha L \Delta T, & \Delta T > \Delta T_y = 3Y/2\alpha E \end{cases}$$

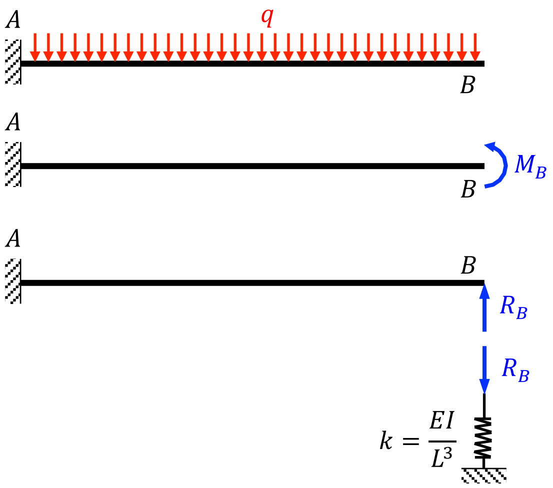
Para $\Delta T = 2\Delta T_y = 3Y/\alpha E$

(c) $\delta(2\Delta T_y) = \dfrac{11YL}{2E}$

Problema 3 (3,5 pontos)



$$k = \frac{EI}{L^3}$$



$$\delta_B^{(1)} = \frac{qL^4}{8EI} \qquad \phi_B^{(1)} = \frac{qL^3}{6EI}$$

$$\delta_B^{(2)} = \frac{M_B L^2}{2EI} \qquad \phi_B^{(2)} = \frac{M_B L}{EI}$$

$$\delta_B^{(3)} = \frac{R_B L^3}{3EI} \qquad \phi_B^{(3)} = \frac{R_B L^2}{2EI}$$

$$\delta_B = \frac{R_B}{k} = \frac{R_B L^3}{EI}$$

$$\delta_B = \delta_B^{(1)} - \delta_B^{(2)} - \delta_B^{(3)} \Rightarrow \frac{R_B L^3}{EI} = \frac{qL^4}{8EI} - \frac{M_B L^2}{2EI} - \frac{R_B L^3}{3EI} \Rightarrow \frac{4}{3}R_B L + \frac{1}{2}M_B = \frac{qL^2}{8}$$

$$0 = \phi_B^{(1)} - \phi_B^{(2)} - \phi_B^{(3)} \Rightarrow 0 = \frac{qL^3}{6EI} - \frac{M_B L}{EI} - \frac{R_B L^2}{2EI} \Rightarrow \frac{1}{2}R_B L + M_B = \frac{qL^2}{6}$$

Resolvendo as equações para R_B e M_B, obtém-se:

$$R_B = \frac{qL}{26}$$

$$M_B = \frac{23qL^2}{156}$$

e portanto

$$\delta_B = \frac{R_B L^3}{EI} = \frac{qL^4}{26EI}$$