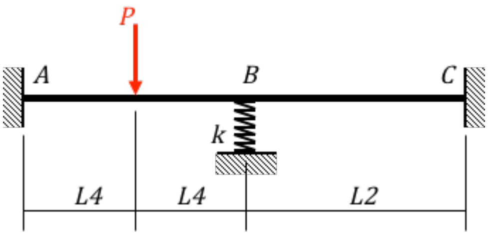
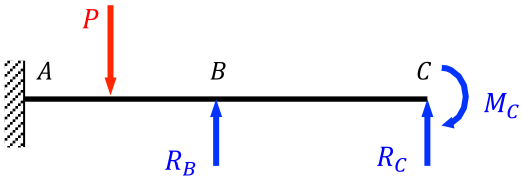
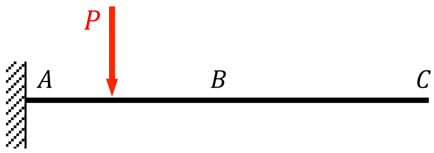
Problema 1 (3,5 pontos).



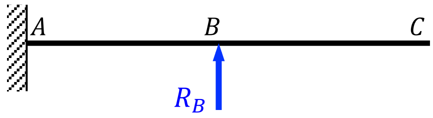




$$\delta_c^{(1)} = \frac{11PL^3}{384EI}$$

$$\phi_c^{(1)} = \frac{PL^2}{32EI}$$

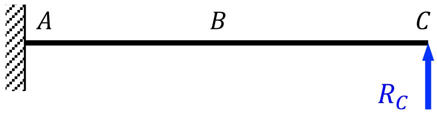
$$\delta_B^{(1)} = \frac{5PL^3}{384EI}$$



$$\delta_c^{(2)} = \frac{5R_BL^3}{48EI}$$

$$\phi_c^{(2)} = \frac{R_BL^2}{8EI}$$

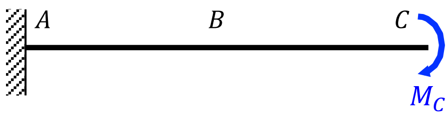
$$\delta_B^{(2)} = \frac{R_BL^3}{24EI}$$



$$\delta_c^{(3)} = \frac{R_CL^3}{3EI}$$

$$\phi_c^{(3)} = \frac{R_CL^2}{2EI}$$

$$\delta_B^{(3)} = \frac{5R_CL^3}{48EI}$$



$$\delta_c^{(4)} = \frac{M_CL^2}{2EI}$$

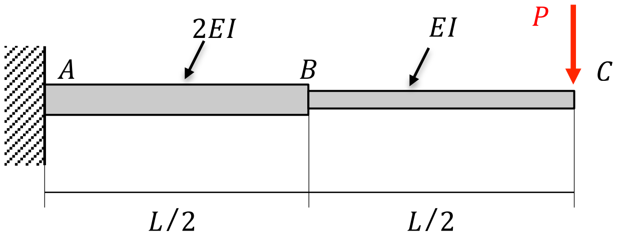
$$\phi_c^{(4)} = \frac{M_CL}{EI}$$

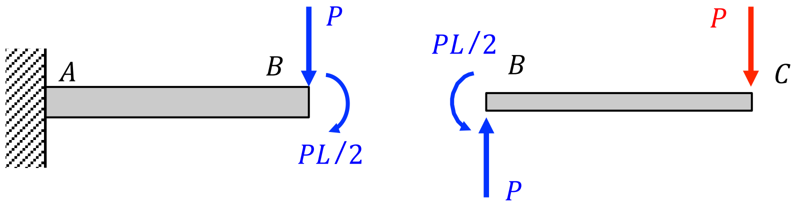
$$\delta_B^{(4)} = \frac{M_CL^2}{8EI}$$

$$\begin{cases} \delta_C = \delta_C^{(1)} - \delta_C^{(2)} - \delta_C^{(3)} + \delta_C^{(4)} = 0 \\ \phi_C = \phi_C^{(1)} - \phi_C^{(2)} - \phi_C^{(3)} + \phi_C^{(4)} = 0 \\ \delta_B = \delta_B^{(1)} - \delta_B^{(2)} - \delta_B^{(3)} + \delta_B^{(4)} = \dfrac{R_B}{k} \end{cases} \Rightarrow \begin{cases} \dfrac{11PL^3}{384EI} - \dfrac{5R_BL^3}{48EI} - \dfrac{R_CL^3}{3EI} + \dfrac{M_CL^2}{2EI} = 0 \\ \dfrac{PL^2}{32EI} - \dfrac{R_BL^2}{8EI} - \dfrac{R_CL^2}{2EI} + \dfrac{M_CL}{EI} = 0 \\ \dfrac{5PL^3}{384EI} - \dfrac{R_BL^3}{24EI} - \dfrac{5R_CL^3}{48EI} + \dfrac{M_CL^2}{8EI} = \dfrac{R_B}{k} \end{cases}$$

$$\begin{cases} \dfrac{11}{384}P - \dfrac{5}{48}R_B - \dfrac{1}{3}R_C + \dfrac{1}{2}\left(\dfrac{M_C}{L}\right) = 0 \\ \dfrac{1}{32}P - \dfrac{1}{8}R_B - \dfrac{1}{2}R_C + \left(\dfrac{M_C}{L}\right) = 0 \\ \dfrac{5}{384}P - \dfrac{1}{24}R_B - \dfrac{5}{48}R_C + \dfrac{1}{8}\left(\dfrac{M_C}{L}\right) = \dfrac{R_B}{(kL^3/EI)} \end{cases} \Rightarrow \boxed{R_B = \left[1 + 64\left(\dfrac{3EI}{kL^3}\right)\right]^{-1}\dfrac{P}{2}}$$

Problema 2 (3,0 pontos)





$$\delta_B = \frac{P(L/2)^3}{3(2EI)} + \frac{(PL/2)(L/2)^2}{2(2EI)} = \frac{5PL^3}{96EI}$$

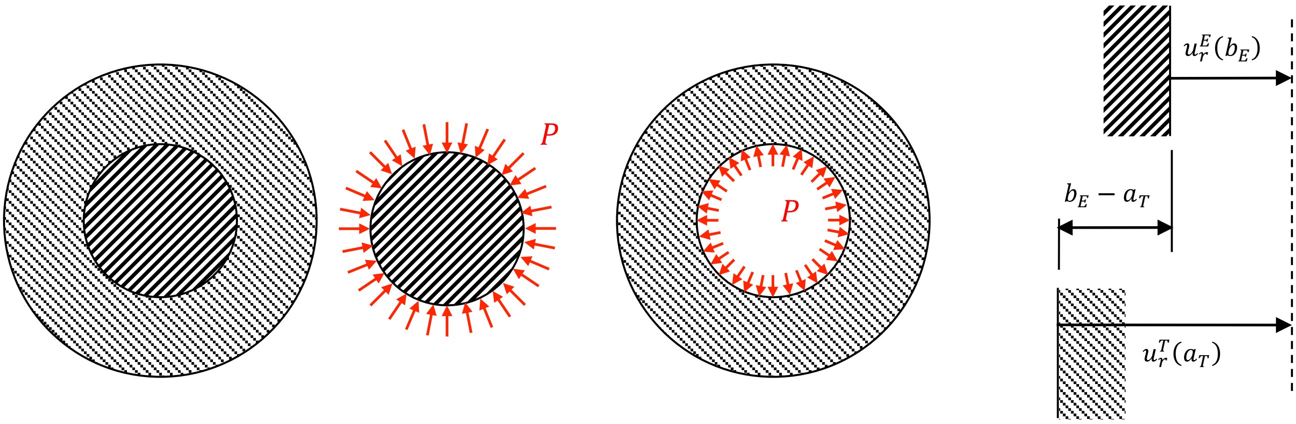
$$\phi_B = \frac{P(L/2)^2}{2(2EI)} + \frac{(PL/2)(L/2)}{(2EI)} = \frac{3PL^2}{16EI}$$

$$\delta_C = \delta_B + \phi_B\frac{L}{2} + \frac{P(L/2)^3}{3(EI)} = \frac{3PL^3}{16EI}$$

$$\phi_C = \phi_B + \frac{P(L/2)^2}{2(EI)} = \frac{5PL^2}{16EI}$$

Problema 3 (3,5 pontos)

(a) 2,0 pontos



$$u_r^T(a_T) = u_r^E(b_E) + (b_E - a_T)$$

$$u_r^E(b_E) = -\frac{(1-\nu)}{E}b_E P$$

$$u_r^T(a_T) = \frac{(1-\nu)a_T^3 + (1+\nu)a_T b_T^2}{E(b_T^2 - a_T^2)}P$$

$$P = \frac{E(b_T^2 - a_T^2)(b_E - a_T)}{(1-\nu)a_T^3 + (1+\nu)a_T b_T^2 + (1-\nu)(b_T^2 - a_T^2)b_E} = 55{,}7\ \text{MPa}$$

(b) 1,5 pontos

Tensões na parede interna do tubo:

$$\sigma_{rr}(a_T) = -P = -55{,}7\ \mathrm{MPa}$$

$$\sigma_{\theta\theta}(a_T) = \frac{\left(\frac{b_T^2}{a_T^2} + 1\right)}{\left(\frac{b_T^2}{a_T^2} - 1\right)} P = 144\ \mathrm{MPa}$$

$$\sigma_{x\theta}(a_T) = a_T \frac{T}{J} = a_T \frac{T}{(\pi b_T^4/2)} = 101\ \mathrm{MPa}$$

Tensor de Tensões:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{x\theta} & \sigma_{xr} \\ \sigma_{x\theta} & \sigma_{\theta\theta} & \sigma_{\theta r} \\ \sigma_{xr} & \sigma_{\theta r} & \sigma_{rr} \end{bmatrix} = \begin{bmatrix} 0 & 101 & 0 \\ 101 & 144 & 0 \\ 0 & 0 & -55{,}7 \end{bmatrix} \mathrm{MPa}$$

No plano $x\theta$

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 72{,}2\ \mathrm{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 124\ \mathrm{MPa}$$

$$\sigma_I = \sigma_m + R = 196\ \mathrm{MPa}$$

$$\sigma_{II} = \sigma_m - R = -51{,}6\ \mathrm{MPa}$$

As tensões principais são ($\sigma_1 > \sigma_2 > \sigma_3$):

$$\sigma_1 = \sigma_I = 196\ \mathrm{MPa}$$

$$\sigma_2 = \sigma_{II} = -51{,}6\ \mathrm{MPa}$$

$$\sigma_3 = \sigma_{rr} = -55{,}7\ \mathrm{MPa}$$

A tensão cisalhante máxima é:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 126\ \mathrm{MPa}$$