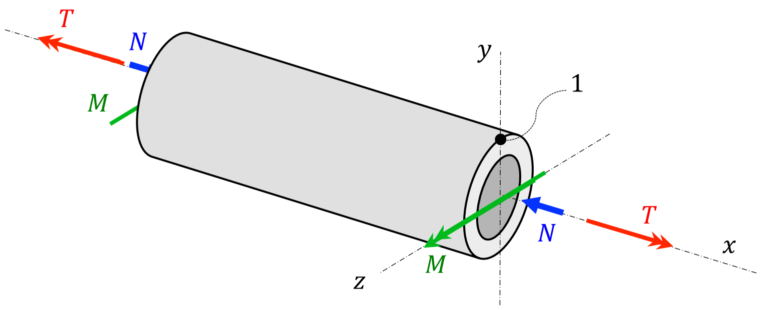
MECÂNICA DOS SÓLIDOS I

P2 – 23/06/2017 – Gabarito

Problema 1 (3,0 pontos).



$$A = \frac{\pi}{4}(D_E^2 - D_I^2) = 2{,}32\times10^{-3}\ \mathrm{m}^2$$

$$J = \frac{\pi}{32}(D_E^4 - D_I^4) = 6{,}73\times10^{-3}\ \mathrm{m}^4$$

$$I = \frac{\pi}{64}(D_E^4 - D_I^4) = 3{,}36\times10^{-3}\ \mathrm{m}^4$$

No Ponto 1 identificado na figura, as tensões normais devido ao esforço normal e momento fletor são ambas negativas, portanto a análise deve ser realizada para o estado de tensão neste ponto:

$$\sigma_{xx} = -\frac{N}{A} - \frac{D_E}{2}\frac{M}{I} = -255\ \mathrm{MPa}$$

$$\sigma_{xz} = \frac{D_E}{2}\frac{T}{J} = 84{,}9\ \mathrm{MPa}$$

As tensões principais no plano xz são:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{zz}}{2} = -128\ \mathrm{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \sigma_{xz}^2} = 153\ \mathrm{MPa}$$

$$\sigma_I = \sigma_m + R = 25{,}7\ \mathrm{MPa}$$

$$\sigma_{II} = \sigma_m - R = -281\ \mathrm{MPa}$$

Ordenando as tensões principais ($\sigma_1 > \sigma_2 > \sigma_3$):

$$\sigma_1 = \sigma_I = 25{,}7\ \mathrm{MPa}$$

$$\sigma_2 = \sigma_{zz} = 0$$

$$\sigma_3 = \sigma_{II} = -281\ \mathrm{MPa}$$

A tensão cisalhante máxima é:

$$\tau_{\mathrm{Max}} = \frac{\sigma_1 - \sigma_3}{2} = 153\ \mathrm{MPa}$$

Empregando o critério de Tresca ($\tau_{\mathrm{Max}} < S_Y/2n_S$):

$$S_Y > 2n_S\tau_{\mathrm{Max}} = 767\ \mathrm{MPa}$$

Problema 2 (3,0 pontos).

As medidas dos estensômetros são:

$$\epsilon(0°) = \epsilon_{xx}$$

$$\epsilon(45°) = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \frac{\epsilon_{xx} - \epsilon_{yy}}{2}\cos 90^\circ + \epsilon_{xy}\sin 90^\circ = \frac{\epsilon_{xx} + \epsilon_{yy}}{2} + \epsilon_{xy}$$

$$\epsilon(90°) = \epsilon_{yy}$$

Portanto:

$$\epsilon_{xx} = \epsilon(0°) = 1{,}10\times10^{-3}$$

$$\epsilon_{xy} = \epsilon(45°) - \frac{\epsilon(0°) + \epsilon(90°)}{2} = -5{,}25\times10^{-4}$$

$$\epsilon_{yy} = \epsilon(90°) = 1{,}80\times10^{-3}$$

As tensões no plano xy (estado plano de tensões) são:

$$\sigma_{xx} = \frac{E}{1-\nu^2}\left(\epsilon_{xx} + \nu\epsilon_{yy}\right) = 137\ \text{MPa}$$

$$\sigma_{yy} = \frac{E}{1-\nu^2}\left(\nu\epsilon_{xx} + \epsilon_{yy}\right) = 175\ \text{MPa}$$

$$\sigma_{xy} = \frac{E}{1+\nu}\epsilon_{xy} = -28{,}4\ \text{MPa}$$

As tensões principais no plano xy são:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy}}{2} = 156\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = 34{,}2\ \text{MPa}$$

$$\sigma_I = \sigma_m + R = 190\ \text{MPa}$$

$$\sigma_{II} = \sigma_m - R = 122\ \text{MPa}$$

Ordenando as tensões principais ($\sigma_1 > \sigma_2 > \sigma_3$):

$$\sigma_1 = \sigma_I = 190\ \text{MPa}$$

$$\sigma_2 = \sigma_{II} = 122\ \text{MPa}$$

$$\sigma_3 = \sigma_{zz} = 0$$

A tensão cisalhante máxima e a tensão de von Mises são:

$$\tau_{\text{Max}} = \frac{\sigma_1 - \sigma_3}{2} = 95{,}0\ \text{MPa}$$

$$\sigma_{\text{VM}} = \sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2\right]} = 167\ \text{MPa}$$

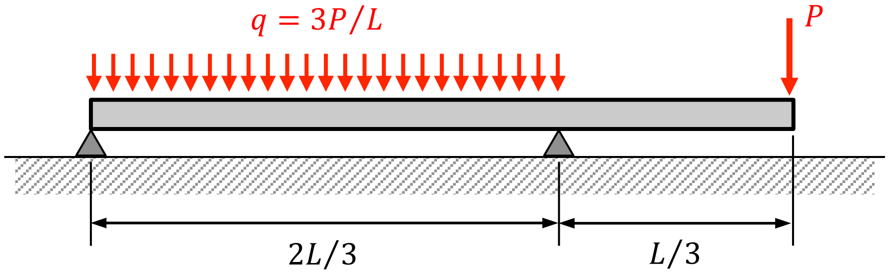
Empregando os critérios de Tresca e von Mises, os coeficientes de segurança são:

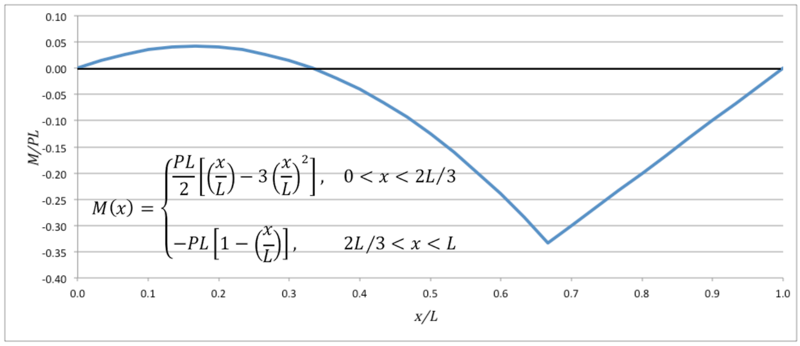
$$n_{\text{Tresca}} = \frac{S_Y}{2\tau_{\text{Max}}} = 2{,}00$$

$$n_{\text{VM}} = \frac{S_Y}{\sigma_{\text{VM}}} = 2{,}28$$

Problema 3 (2,0 pontos).

A figura abaixo apresenta a distribuição de momentos fletores:





O momento fletor máximo em valor absoluto ocorre no apoio da direita, em $x = 2L/3$:

$$\max\{|M(x)|\} = |M(2L/3)| = PL/3$$

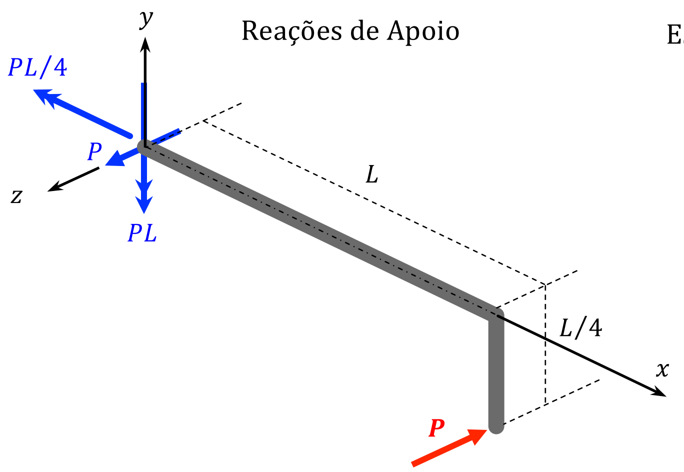
Logo a tensão normal máxima é:

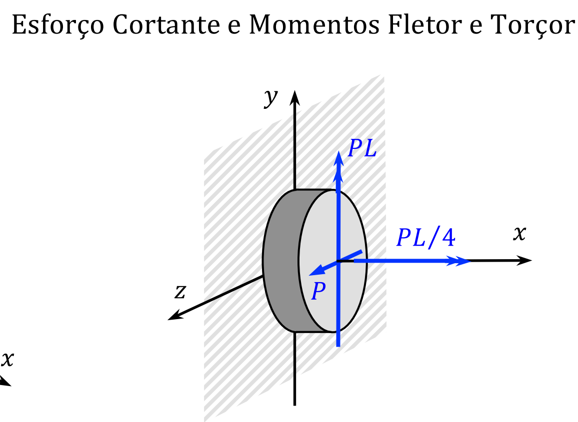
$$\max\{|\sigma_{xx}(x,y)|\} = \max\{|y|\}\frac{\max\{|M(x)|\}}{I} = \left(\frac{h}{2}\right)\frac{(PL/3)}{(bh^3/12)} = \frac{2PL}{bh^2}$$

Portanto:

$$\max\{|\sigma_{xx}(x,y)|\} < S_Y \quad \Rightarrow \quad P < \frac{bh^2 S_Y}{2L}$$

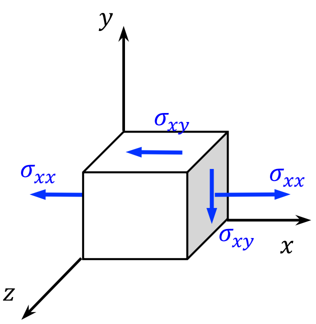
Problema 4 (2,0 pontos).





$$\sigma_{xx} = \left(\frac{D}{2}\right)\frac{(PL)}{(\pi D^4/64)} = \frac{32PL}{\pi D^3}$$

$$\sigma_{xy} = -\left(\frac{D}{2}\right)\frac{(PL/4)}{(\pi D^4/32)} = -\frac{4PL}{\pi D^3}$$



$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ & \sigma_{yy} & \sigma_{yz} \\ & & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \frac{32PL}{\pi D^3} & -\frac{4PL}{\pi D^3} & 0 \\ & 0 & 0 \\ & & 0 \end{bmatrix}$$