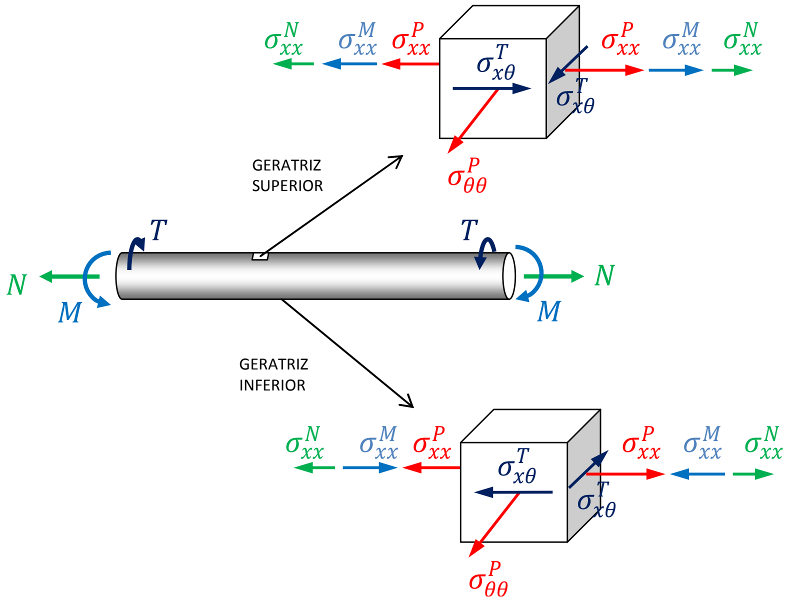
Problema 1 (3,5 pontos)

Para análise devem ser considerados os pontos da geratriz superior do tubo, onde a tensão axial trativa (positiva) devido à flexão soma-se às tensões axiais produzidas pelo esforço normal e pressão interna, também positivas.



(a) Tensões devido à pressão interna

$$\sigma_{\theta\theta}^P = \frac{pD}{2t} = 126 \text{ MPa}$$

$$\sigma_{xx}^P = \frac{pD}{4t} = 63{,}0 \text{ MPa}$$

(b) Tensão axial devido à Flexão

$$\sigma_{xx}^M = \left(\frac{D}{2}\right)\frac{M}{(\pi D^3 t/8)} = 108 \text{ MPa}$$

(c) Tensão axial devido ao esforço normal

$$\sigma_{xx}^N = \frac{N}{\pi D t} = 5{,}14 \text{ MPa}$$

(d) Tensão cisalhante devido à torção

$$\sigma_{x\theta}^T = \left(\frac{D}{2}\right)\frac{T}{(\pi D^3 t/4)} = 162\ MPa$$

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{x\theta} & \sigma_{xr} \\ \sigma_{x\theta} & \sigma_{\theta\theta} & \sigma_{\theta r} \\ \sigma_{xr} & \sigma_{\theta r} & \sigma_{rr} \end{bmatrix} = \begin{bmatrix} \sigma_{xx}^P + \sigma_{xx}^M + \sigma_{xx}^N & \sigma_{x\theta}^T & 0 \\ \sigma_{x\theta}^T & \sigma_{\theta\theta}^P & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 176 & 162 & 0 \\ 162 & 126 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{MPa}$$

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 151 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 164 \text{ MPa}$$

$$\sigma_I = \sigma_m + R = 315 \text{ MPa}$$

$$\sigma_{II} = \sigma_m - R = -12{,}8 \text{ MPa}$$

$$\sigma_1 = \sigma_I = 315 \text{ MPa}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \sigma_{II} = -12{,}8 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 164 \text{ MPa}$$

$$\sigma_{VM} = \sqrt{\frac{(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2}{2}} = 322 \text{ MPa}$$

$$n_{\text{Tresca}} = \frac{S_Y}{2\tau_{max}} = 1{,}50$$

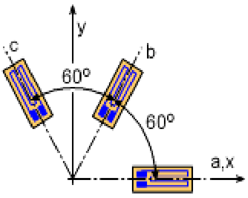
$$n_{VM} = \frac{S_Y}{\sigma_{VM}} = 1{,}52$$

Problema2 (3,5 pontos)

Para a roseta extensométrica a 60°:

$$\left.\begin{aligned}\varepsilon_a &= \varepsilon(0°) = \varepsilon_{xx}\\ \varepsilon_b &= \varepsilon(60°) = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}+\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos 120° + \varepsilon_{xy}\sin 120° = \frac{\varepsilon_{xx}}{4}+\frac{3\varepsilon_{yy}}{4}+\frac{\sqrt{3}}{2}\varepsilon_{xy}\\ \varepsilon_c &= \varepsilon(120°) = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2}+\frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}cos\,240° + \varepsilon_{xy}\,sin\,240° = \frac{\varepsilon_{xx}}{4}+\frac{3\varepsilon_{yy}}{4}-\frac{\sqrt{3}}{2}\varepsilon_{xy}\end{aligned}\right\} \Rightarrow \left\{\begin{aligned}\varepsilon_{xx} &= \varepsilon_a\\ \varepsilon_{yy} &= \frac{2(\varepsilon_b+\varepsilon_c)-\varepsilon_a}{3}\\ \varepsilon_{xy} &= \frac{\varepsilon_b-\varepsilon_c}{\sqrt{3}}\end{aligned}\right.$$

$$\varepsilon_{xx} = 600 \times 10^{-6}$$



$$\varepsilon_{yy} = 1100 \times 10^{-6}$$

$$\varepsilon_{xy} = -814 \times 10^{-6}$$

$$\sigma_{xx} = \frac{E}{1-\nu^2}\left(\varepsilon_{xx}+\nu\,\varepsilon_{yy}\right) = 204\ \text{MPa}$$

$$\sigma_{yy} = \frac{E}{1-\nu^2}\left(\nu\,\varepsilon_{xx}+\varepsilon_{yy}\right) = 281\ \text{MPa}$$

$$\sigma_{xy} = \frac{E}{1+\nu}\varepsilon_{xy} = -125\ \text{MPa}$$

$$\sigma_m = \frac{\sigma_{xx}+\sigma_{yy}}{2} = 243\ \text{MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = 131\ \text{MPa}$$

$$\sigma_I = \sigma_m + R = 374\ \text{MPa}$$

$$\sigma_{II} = \sigma_m - R = 112\ \text{MPa}$$

$$\sigma_1 = \sigma_I = 374\ \text{MPa}$$

$$\sigma_2 = \sigma_{II} = 112\ MPa$$

$$\sigma_3 = 0$$

Problema 3 (3,0 pontos)

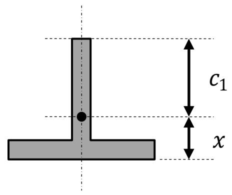
Para as vigas engastadas:

$\max\{M(x)\} = M(0) = qL^2/2$

$$\max\{\sigma_{xx}(x,y)\} = c_1 \frac{\max\{M(x)\}}{I} = \frac{qL^2/2}{I/c_1} < \frac{S_Y}{n} \Rightarrow W = I/c_1 > \frac{nqL^2}{2S_Y} = 0{,}68 \text{ cm}^3$$

Logo, o perfil escolhido para as três vigas é o de bitola 1¼” × ⅛” com $W = I/c_1 = 0{,}81 \text{ cm}^3$

Bitola	Mesa	Espessura	Peso	Área	Eixo X			
	$d=b_f$	$t_f=t_w$	Nominal		I	W	r	x
	mm	mm	kg/m	cm^2	cm^4	cm^3	cm	cm
3/4" serr.	19,05	2,50	0,69					
5/8 x 1/8"	15,88	3,18	0,71	0,90	0,20	0,19	0,47	0,51
3/4 x 1/8"	19,05	3,18	0,86	1,13	0,36	0,27	0,57	0,59
7/8 x 1/8"	22,22	3,18	0,99	1,34	0,59	0,38	0,67	0,67
1 x 1/8"	25,40	3,18	1,18	1,54	0,90	0,50	0,77	0,75
1.1/4 x 1/8"	31,75	3,18	1,50	1,92	1,84	0,81	0,98	0,91
1.1/2 x 1/8"	38,10	3,18	1,82	2,32	3,24	1,18	1,18	1,07
1.1/4 x 3/16"	31,75	4,76	2,16	2,79	2,56	1,16	0,96	0,97
1.1/2 x 3/16"	38,10	4,76	2,65	3,40	4,56	1,70	1,16	1,13
2 x 3/16"	50,80	4,76	3,62	4,61	11,33	3,12	1,57	1,45
2 x 1/4"	50,80	6,35	4,74	6,05	14,47	4,04	1,55	1,50



$c_1 > x$

$W = I/c_1$

Fonte: *Gerdau - Catálogo de Barras e Perfis* (www.comercialgerdau.com.br)

Para a viga simplesmente apoiada

$\max\{M(x)\} = M(L/2) = PL/4$

$$\max\{\sigma_{xx}(x,y)\} = c_1 \frac{\max\{M(x)\}}{I} = \frac{PL/4}{I/c_1} < \frac{S_Y}{n} \Rightarrow L < \frac{4(I/c_1)S_Y}{nP} = \frac{4WS_Y}{nP} = 2{,}70 \text{ m}$$

O comprimento da viga simplesmente apoiada deve portanto ser $L = 2{,}70$ m.