Problema 1.

(i) Tensões devido à pressão interna

$$\sigma_{xx}(x,y) = pD/4t$$
$$\sigma_{\theta\theta}(x,y) = pD/2t$$

(ii) Tensões devido à flexão

$$M(x) = \frac{qL^2}{2}\left[\left(\frac{x}{L}\right) - \left(\frac{x}{L}\right)^2\right], \quad \max\{|M(x)|\} = M(L/2) = \frac{qL^2}{8}$$

(ii-1) Na face superior da viga ($x = L/2,\ y = D/2$)

$$\sigma_{xx}(L/2, D/2) = -\left(\frac{D}{2}\right)\frac{(qL^2/8)}{(\pi D^3 t/8)} = -\frac{qL^2}{2\pi D^2 t}$$

(ii-1) Na face inferior da viga ($x = L/2,\ y = -D/2$)

$$\sigma_{xx}(L/2, -D/2) = -\left(-\frac{D}{2}\right)\frac{(qL^2/8)}{(\pi D^3 t/8)} = \frac{qL^2}{2\pi D^2 t}$$

(iii) Tensões combinadas

(iii-1) Na face superior ($x = L/2,\ y = D/2$)

$$\sigma_{xx}(L/2, D/2) = pD/4t - qL^2/2\pi D^2 t$$
$$\sigma_{zz}(L/2, D/2) = pD/2t$$

$$\left.\begin{array}{l}\sigma_1 = \sigma_{zz}(L/2, D/2) = pD/2t \\ \sigma_2 = \sigma_{xx}(L/2, D/2) = pD/4t - qL^2/2\pi D^2 t \geq 0 \\ \sigma_3 = 0\end{array}\right\} \Rightarrow \quad \tau_{\max} = pD/4t = 50 \text{ MPa} \leq \tau_Y \text{ (ok)}$$

$$\left.\begin{array}{l}\sigma_1 = \sigma_{zz}(L/2, D/2) = pD/2t \\ \sigma_2 = 0 \\ \sigma_3 = \sigma_{xx}(L/2, D/2) = pD/4t - qL^2/2\pi D^2 t < 0\end{array}\right\} \Rightarrow \quad \tau_{\max} = pD/8t + qL^2/4\pi D^2 t \leq \tau_Y$$

(iii-1) Na face inferior ($x = L/2,\ y = -D/2$)

$$\sigma_{xx}(L/2, -D/2) = pD/4t + qL^2/2\pi D^2 t$$
$$\sigma_{zz}(L/2, -D/2) = pD/2t$$

$$\left.\begin{array}{l}\sigma_1 = \sigma_{zz}(L/2, -D/2) = pD/2t \\ \sigma_2 = \sigma_{xx}(L/2, -D/2) = pD/4t + qL^2/2\pi D^2 t < pD/2t \\ \sigma_3 = 0\end{array}\right\} \Rightarrow \tau_{\max} = pD/4t = 50 \text{ MPa} \leq \tau_Y \quad \text{(ok)}$$

$$\left.\begin{array}{l}\sigma_1 = \sigma_{xx}(L/2, D/2) = pD/4t + qL^2/2\pi D^2 t > pD/2t \\ \sigma_2 = \sigma_{zz}(L/2, D/2) = pD/2t \\ \sigma_3 = 0\end{array}\right\} \quad \Rightarrow \quad \tau_{\max} = pD/8t + qL^2/4\pi D^2 t \le \tau_{\mathrm{Y}}$$

$$\tau_{\max} = pD/8t + qL^2/4\pi D^2 t \le \tau_{\mathrm{Y}} \quad \Rightarrow \quad L \le \sqrt{(4\pi D^2 t/q)(\tau_{\mathrm{Y}} - pD/8t)} = 12{,}0 \text{ m}$$

Problema 2.

$$\varepsilon_A = 360\times10^{-6}, \quad \varepsilon_B = 165\times10^{-6}, \quad \varepsilon_B = -290\times10^{-6}$$

$$\varepsilon_A = \varepsilon_{xx}$$

$$\varepsilon_B = \varepsilon(45°) = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \frac{\varepsilon_{xx}-\varepsilon_{yy}}{2}\cos(90°) + \varepsilon_{xy}\sin(90°) = \frac{\varepsilon_{xx}+\varepsilon_{yy}}{2} + \varepsilon_{xy}$$

$$\varepsilon_C = \varepsilon_{yy}$$

$$\varepsilon_{xx} = \varepsilon_A = 360\times10^{-6}$$

$$\varepsilon_{yy} = \varepsilon_C = -165\times10^{-6}$$

$$\varepsilon_{xy} = \frac{2\varepsilon_B - (\varepsilon_A + \varepsilon_C)}{2} = 130\times10^{-6}$$

$$\left.\begin{array}{l}\varepsilon_{xx} = \dfrac{\sigma_{xx}}{E} - \nu\dfrac{\sigma_{yy}}{E} \\ \varepsilon_{yy} = -\nu\dfrac{\sigma_{xx}}{E} + \dfrac{\sigma_{yy}}{E}\end{array}\right\} \quad \Rightarrow \quad \left\{\begin{array}{l}\sigma_{xx} = \dfrac{E}{1-\nu^2}(\varepsilon_{xx} + \nu\,\varepsilon_{yy}) = 60\,\mathrm{MPa} \\ \sigma_{yy} = \dfrac{E}{1-\nu^2}(\nu\,\varepsilon_{yy} + \varepsilon_{xx}) = -40\,\mathrm{MPa}\end{array}\right.$$

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xy} \quad \Rightarrow \quad \sigma_{xy} = \frac{E}{1+\nu}\varepsilon_{xy} = 20\,\mathrm{MPa}$$

$$\sigma_m = \frac{\sigma_{xx}+\sigma_{yy}}{2} = 10\,\mathrm{MPa}, \quad \mathrm{R} = \sqrt{\left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = 53{,}9\,\mathrm{MPa}$$

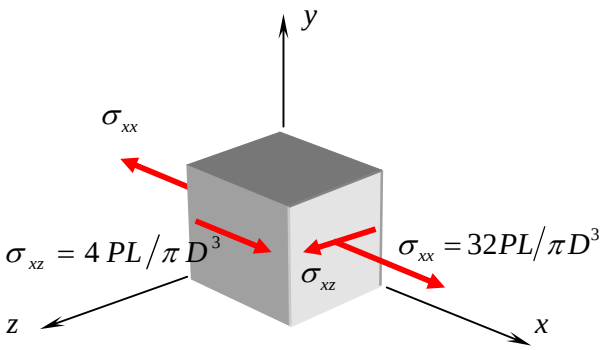
$$\sigma_{\mathrm{I}} = \sigma_m + R = 63{,}9 \text{ MPa}, \quad \sigma_{\mathrm{II}} = \sigma_m - R = -43{,}9 \text{ MPa}$$

$$\sigma_1 = \sigma_{\mathrm{I}} = 63{,}9 \text{ MPa}, \quad \sigma_2 = \sigma_{zz} = 0 \quad \text{e} \quad \sigma_3 = \sigma_{II} = -43{,}9 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 53{,}9 \text{ MPa}$$

Problema 3.

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \frac{32PL}{\pi D^3} & 0 & \frac{4PL}{\pi D^3} \\ 0 & 0 & 0 \\ \frac{4PL}{\pi D^3} & 0 & 0 \end{bmatrix}$$



Problema 4.

$M(x) = Px/2 \quad (x < L/2)$

$x = 0.1 \Rightarrow M(0.1) = 0.05P$

$\sigma_{xx}(x,y) = -y\, M(x)/I, \quad I = bh^3/12 = 1.71\times10^{-6}\,\text{m}^4$

$\sigma_{xx}(0.1,-0.02) = -(-0.02)\,(0.05P)/(1{,}71\times10^{-6}) = 586P$

$\sigma_{xx} = E\varepsilon_{xx} \quad \Rightarrow \quad 586P = (200\times10^9)(645\times10^{-6}) \quad \Rightarrow \quad P = 220\,\text{kN}$