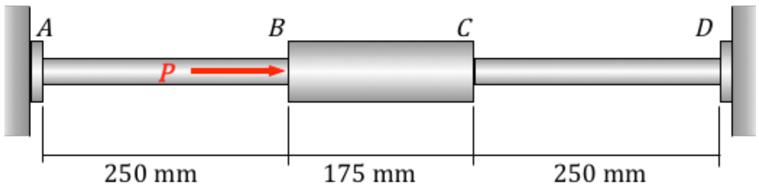
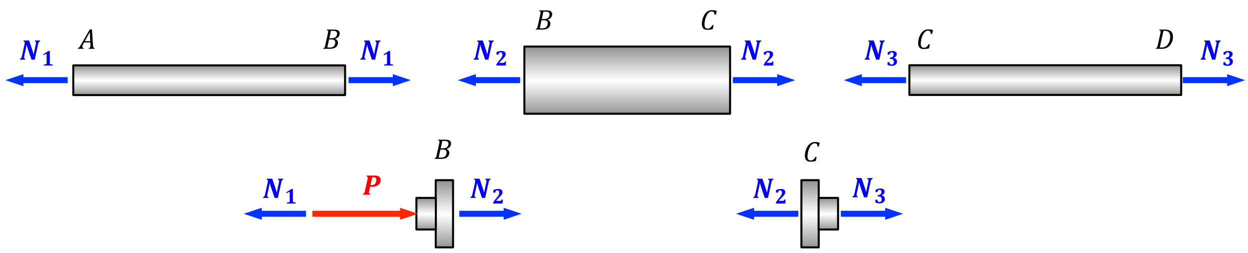
MECÂNICA DOS SÓLIDOS I
Gabarito – P1 – 05/05/2017

Problema 1 (3,0 pontos).





$$N_1 - N_2 = P$$

$$N_2 - N_3 = 0$$

$$u_B - u_A = C_1 N_1, \quad C_1 = L_1/E_1A_1$$

$$u_C - u_B = C_2 N_2, \quad C_2 = L_2/E_2A_2$$

$$u_D - u_C = C_3 N_3, \quad C_3 = L_3/E_3A_3$$

$$C_1 = C_3 = 2{,}50\times10^{-9}\ \mathrm{m/N}$$

$$C_2 = 1{,}25\times10^{-9}\ \mathrm{m/N}$$

$$u_A = u_D = 0$$

$$-C_3N_3 - C_1N_1 = C_2N_2$$

$$N_1 = \frac{C_2 + C_3}{C_1 + C_2 + C_3}P = 60\ \mathrm{kN}$$

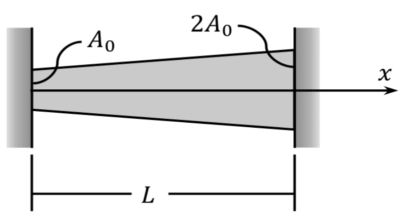
$$N_2 = N_3 = -\frac{C_1}{C_2 + C_3}N_1 = -\frac{C_1}{C_1 + C_2 + C_3}P = -40\ \mathrm{kN}$$

$$\sigma^{(1)} = \frac{N_1}{A_1} = 120\ \mathrm{MPa}$$

$$\sigma^{(2)} = \frac{N_2}{A_2} = -57{,}1\ \mathrm{MPa}$$

$$\sigma^{(3)} = \frac{N_3}{A_3} = -80{,}0\ \mathrm{MPa}$$

Problema 2 (2,0 pontos).



$$\frac{dN}{dx} = 0 \Rightarrow N = \mathrm{cte.}$$

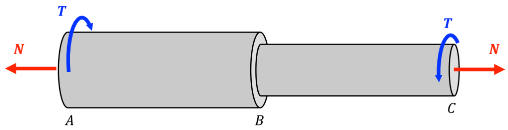
$$\frac{du}{dx} = \frac{N}{EA(x)} + \alpha\Delta T, \quad A(x) = \frac{A_0}{L}(L + x)$$

$$u(L) - u(0) = \frac{NL}{EA_0}\int_0^L \frac{dx}{L + x} + \alpha L\Delta T = \frac{NL}{EA_0}\ln(2) + \alpha L\Delta T = 0$$

$$N = -\frac{\alpha E A_0 \Delta T}{\ln(2)} = -1{,}44\alpha E A_0 \Delta T$$

$$\max\{|\sigma_{xx}|\} = \frac{|N|}{\min\{A(x)\}} = \frac{\alpha E \Delta T}{\ln(2)}$$

Problema 3 (3,0 pontos).



(a) Segmento AB

$$\sigma_{xx} = \frac{N}{A_1} = 78{,}4 \text{ MPa}$$

$$\sigma_{x\theta} = \frac{D_1}{2}\frac{T}{J_1} = 47{,}0 \text{ MPa}$$

$$\sigma_{\theta\theta} = 0$$

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 39{,}2 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 - \sigma_{x\theta}^2} = 61{,}2 \text{ MPa}$$

$$\sigma_1 = \sigma_I = \sigma_m + R = 100 \text{ MPa}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \sigma_{II} = \sigma_m - R = -22{,}0 \text{ MPa}$$

(a) Segmento BC

$$\sigma_{xx} = \frac{N}{A_2} = 122 \text{ MPa}$$

$$\sigma_{x\theta} = \frac{D_2}{2}\frac{T}{J_2} = 163 \text{ MPa}$$

$$\sigma_{\theta\theta} = 0$$

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 61{,}1 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 - \sigma_{x\theta}^2} = 174 \text{ MPa}$$

$$\sigma_1 = \sigma_I = \sigma_m + R = 235 \text{ MPa}$$

$$\sigma_2 = 0$$

$$\sigma_3 = \sigma_{II} = \sigma_m - R = -113 \text{ MPa}$$

Problema 4 (2,0 pontos).

(a) $$[\sigma] = \begin{bmatrix} 0 & \tau & 0 \\ \tau & 0 & -\tau \\ 0 & -\tau & 0 \end{bmatrix}$$

$$\det\begin{pmatrix} -\lambda & \tau & 0 \\ \tau & -\lambda & -\tau \\ 0 & -\tau & -\lambda \end{pmatrix} = -\lambda^3 + 2\tau^2\lambda = 0$$

$$\sigma_1 = \tau\sqrt{2},\ \ \sigma_2 = 0,\ \ e\ \ \sigma_3 = -\tau\sqrt{2}$$

(b) $$[\sigma] = \begin{bmatrix} s/2 & s/2 & 0 \\ s/2 & s/2 & 0 \\ 0 & 0 & s \end{bmatrix}$$

No plano xy:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy}}{2} = s/2$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 - \sigma_{xy}^2} = s/2$$

$$\sigma_I = \sigma_m + R = s$$

$$\sigma_{II} = \sigma_m - R = 0$$

Logo

$$\sigma_1 = \sigma_I = s,\ \ \sigma_2 = \sigma_{xx} = s,\ \ e\ \ \sigma_3 = \sigma_{II} = 0$$