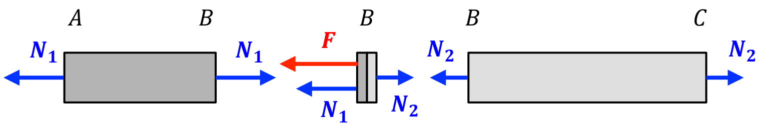
Problema 1 (2,0 pontos).



$$N_2 - N_1 = F$$

$$u_B - u_A = \frac{N_1 L_1}{E_1 A_1} + \alpha_1 L_1 \Delta T = \frac{L}{3EA} N_1 + 2\alpha L \Delta T$$

$$u_C - u_B = \frac{N_2 L_2}{E_1 A_1} + \alpha_2 L_2 \Delta T = \frac{2L}{EA} N_2 + 2\alpha L \Delta T$$

$$u_A = u_C = 0$$

$$u_B = \frac{L}{3EA} N_1 + 2\alpha L \Delta T = -\frac{2L}{EA} N_2 - 2\alpha L \Delta T$$

$$N_1 + 6N_2 = -12\alpha EA\Delta T$$

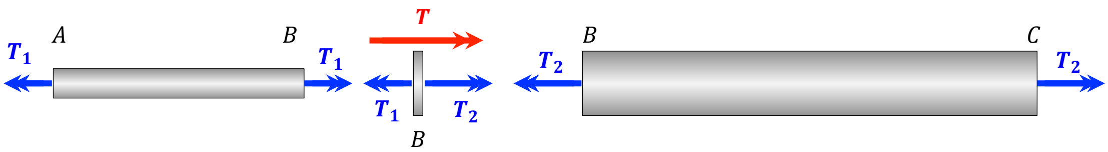
(a) $F \neq 0$ e $\Delta T = 0$

$$\begin{cases} N_2 - N_1 = F \\ N_1 + 6N_2 = 0 \end{cases} \Rightarrow \begin{cases} N_1 = -6F/7 \\ N_2 = F/7 \end{cases}$$

(b) $F = 0$ e $\Delta T \neq 0$

$$\begin{cases} N_2 - N_1 = 0 \\ N_1 + 6N_2 = -12\alpha EA\Delta T \end{cases} \Rightarrow N_1 = N_2 = -12\alpha EA\Delta T/7$$

Problema 2 (2,0 pontos).



$$T_1 - T_2 = T$$

$$\phi_B - \phi_A = \frac{T_1 L_1}{G_1 J_1} = \frac{T_1}{k_1}, \quad k_1 = \frac{G_1 J_1}{L_1} = \frac{\pi G D^4}{32L} T$$

$$\phi_C - \phi_B = \frac{T_2 L_2}{G_2 J_2} = \frac{T_2}{k_2}, \quad k_2 = \frac{G_2 J_2}{L_2} = \frac{\pi G D^4}{4L} T$$

$$\phi_A = \phi_C = 0$$

$$\phi_B = \frac{T_1}{k_1} = -\frac{T_2}{k_2}$$

$$\begin{cases} T_1 - T_2 = T \\ T_2 = -k_2 N_2 / k_1 \end{cases} \Rightarrow \begin{cases} T_1 = k_1 T/(k_1 + k_2) = T/9 \\ T_2 = -k_2 T/(k_1 + k_2) = -8T/9 \end{cases}$$

$$\phi_B = \frac{T_1}{k_1} = \frac{T}{k_1 + k_2} = \frac{32TL}{9\pi G D^4}$$

Problema 3 (2,5 pontos).

(a) $$[\sigma] = \begin{bmatrix} 0 & -40 & 0 \\ -40 & 0 & 20 \\ 0 & 20 & 0 \end{bmatrix} \text{ MPa}$$

$$\det \begin{bmatrix} -\lambda & -40 & 0 \\ -40 & -\lambda & 20 \\ 0 & 20 & -\lambda \end{bmatrix} = -\lambda(\lambda^2 - 2000) = 0 \;\Rightarrow\; \begin{cases} \sigma_1 = 44{,}7 \text{ MPa} \\ \sigma_2 = 0 \\ \sigma_3 = -44{,}7 \text{ MPa} \end{cases}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 44{,}7 \text{ MPa}$$

(b) $$[\sigma] = \begin{bmatrix} 60 & 40 & 0 \\ 40 & 0 & 0 \\ 0 & 0 & -80 \end{bmatrix} \text{ MPa}$$

No plano xy

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy}}{2} = 30 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = \sqrt{30^2 + 40^2} = 50 \text{ MPa}$$

$$\sigma_I = \sigma_m + R = 80 \text{ MPa}$$

$$\sigma_{II} = \sigma_m - R = -20 \text{ MPa}$$

Ordenando as tensões principais ($\sigma_1 > \sigma_2 > \sigma_3$)

$$\sigma_1 = \sigma_I = 80 \text{ MPa}$$

$$\sigma_2 = \sigma_{II} = -20 \text{ MPa}$$

$$\sigma_3 = \sigma_{zz} = -80 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 80 \text{ MPa}$$

Problema 4 (3,5 pontos).

TORÇÃO	**ESFORÇO NORMAL**
$T_{Aço} + T_{Al} = T$ $\Delta\phi_{Aço} = \dfrac{T_{Aço}}{G_{Aço}J_{Aço}/L}$ $\Delta\phi_{Al} = \dfrac{T_{Al}}{G_{Al}J_{Al}/L}$ $\Delta\phi_{Aço} = \Delta\phi_{Al}$ $T_{Aço} = \dfrac{G_{Aço}J_{Aço}}{G_{Aço}J_{Aço} + G_{Al}J_{Al}}T$ $T_{Al} = \dfrac{G_{Al}J_{Al}}{G_{Aço}J_{Aço} + G_{Al}J_{Al}}T$	$N_{Aço} + N_{Al} = N$ $\Delta u_{Aço} = \dfrac{N_{Aço}}{E_{Aço}A_{Aço}/L}$ $\Delta u_{Al} = \dfrac{N_{Al}}{G_{Al}J_{Al}/L}$ $\Delta u_{Aço} = \Delta u_{Al}$ $N_{Aço} = \dfrac{E_{Aço}A_{Aço}}{E_{Aço}A_{Aço} + E_{Al}A_{Al}}N$ $N_{Al} = \dfrac{E_{Al}A_{Al}}{E_{Aço}A_{Aço} + E_{Al}A_{Al}}N$
$G_{Aço}J_{Aço} = 5{,}63\times10^3 \text{ N}\cdot\text{m}^2$ e $G_{Al}J_{Al} = 1{,}29\times10^2 \text{ N}\cdot\text{m}^2$	$E_{Aço}A_{Aço} = 7{,}04\times10^7$ N e $E_{Al}A_{Al} = 1{,}24\times10^7$ N
$T_{Aço} = 293 \text{ N}\cdot\text{m}$ $T_{Al} = 6{,}73 \text{ N}\cdot\text{m}$	$N_{Aço} = 42{,}5$ kN $N_{Al} = 7{,}48$ kN

Tensões máximas no tubo de aço (na superfície externa do tubo):

$$\sigma_{xx} = \frac{N_{Aço}}{A_{Aço}} = 121 \text{ MPa}$$

$$\sigma_{x\theta} = \frac{D_{ext}}{2}\frac{T_{Aço}}{J_{Aço}} = 66{,}7 \text{ MPa}$$

Tensões máximas no eixo de alumínio (na superfície externa do eixo de alumínio):

$$\sigma_{xx} = \frac{N_{Al}}{A_{Al}} = 42{,}3 \text{ MPa}$$

$$\sigma_{x\theta} = \frac{D_{Al}}{2}\frac{T_{Al}}{J_{Al}} = 10{,}2 \text{ MPa}$$

Portanto a máxima tensão cisalhante ocorrerá na superfície externa do tubo de aço. Neste ponto as tensões principais são calculadas por meio de:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 60{,}4 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 90{,}0 \text{ MPa}$$

$$\sigma_I = \sigma_m + R = 150 \text{ MPa}$$

$$\sigma_{II} = \sigma_m - R = -29{,}6 \text{ MPa}$$

Ordenando as tensões principais ($\sigma_1 > \sigma_2 > \sigma_3$)

$$\sigma_1 = \sigma_I = 150 \text{ MPa}$$

$$\sigma_2 = \sigma_{rr} = 0$$

$$\sigma_3 = \sigma_{II} = -29{,}6 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 90{,}0 \text{ MPa}$$