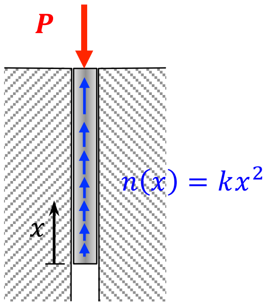
Problema 1 (2,0 pontos)

(a) 0,5 ponto

$$P = \int_0^L kx^2 dx = \frac{kL^3}{3}$$



(b) 0,5 ponto

$$\frac{dN}{dx} + kx^2 = 0 \;\Rightarrow\; N(x) = -\frac{kx^3}{3} + c_1$$

$$N(0) = 0 \;\Rightarrow\; c_1 = 0 \;\Rightarrow\; N(x) = -\frac{kx^3}{3}$$

$$\sigma(x) = \frac{N(x)}{A} = -\frac{kx^3}{3A}$$

Máxima tensão axial em valor absoluto:

$$\max\{|\sigma(x)|\} = |\sigma(L)| = \frac{kL^3}{3A}$$

(c) 1,0 ponto

$$\frac{du}{dx} = \frac{N(x)}{EA} = -\frac{kx^3}{3EA} \;\Rightarrow\; u(x) = -\frac{kx^4}{12EA} + c_2$$

$$u(0) = c_2 \text{ e } u(L) = -\frac{kL^4}{12EA} + c_2$$

$$\Delta u = u(L) - u(0) = -\frac{kL^4}{12EA}$$

Problema 2 (1,5 pontos).

(a) 0,5 ponto

No plano xz:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{zz}}{2} = 0$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \sigma_{xz}^2} = \tau$$

$$\sigma_I = \sigma_m + R = \tau$$

$$\sigma_{II} = \sigma_m - R = -\tau$$

$$\sigma_1 = \sigma_{yy} = 2\tau$$

$$\sigma_2 = \sigma_I = \tau$$

$$\sigma_3 = \sigma_{II} = -\tau$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \frac{3\tau}{2}$$

(b) 0,5 ponto

No plano xz:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{zz}}{2} = \tau$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \sigma_{xz}^2} = \tau$$

$$\sigma_I = \sigma_m + R = 2\tau$$

$$\sigma_{II} = \sigma_m - R = 0$$

$$\sigma_1 = \sigma_I = 2\tau$$

$$\sigma_2 = \sigma_{yy} = \tau$$

$$\sigma_3 = \sigma_{II} = 0$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \tau$$

(b) 0,5 ponto

No plano xy:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{yy}}{2} = 65{,}0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xz}^2} = 53{,}2 \text{ MPa}$$

$$\sigma_I = \sigma_m + R = 118 \text{ MPa}$$

$$\sigma_{II} = \sigma_m - R = 11{,}8 \text{ MPa}$$

$$\sigma_1 = \sigma_I = 118 \text{ MPa}$$

$$\sigma_2 = \sigma_{zz} = 60{,}0 \text{ MPa}$$

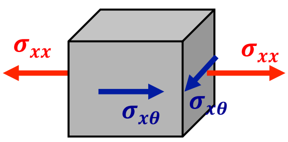
$$\sigma_3 = \sigma_{II} = 11{,}8 \text{ MPa}$$

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 53{,}2 \text{ MPa}$$

Problema 3 (3,0 pontos)

Tensão produzida pelo esforço normal:

$$\sigma_{xx} = \frac{N}{A} = \frac{5.00\times10^4}{4.91\times10^{-4}} = 102 \text{ MPa}$$



Tensão produzida pela torção:

$$\sigma_{x\theta}(r) = r\frac{T}{J}$$

A máxima tensão devido à torção ocorre em $r = D/2$

$$\sigma_{x\theta}(D/2) = \frac{D}{2}\frac{T}{J} = \frac{2.50\times10^{-2}}{2}\frac{T}{3.83\times10^{-8}} = 0.326T$$

onde a tensão é dada em MPa e o torque em $\text{N}\cdot\text{m}$.

Empregando as fórmulas para o estado plano de tensão:

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 51.0 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = \sqrt{(51.0)^2 + (0.326T)^2}$$

$$\sigma_I = \sigma_m + R = 51.0 + \sqrt{(51.0)^2 + (0.326T)^2} > 0$$

$$\sigma_{II} = \sigma_m - R = 51.0 - \sqrt{(51.0)^2 + (0.326T)^2} < 0$$

Logo, ordenando-se as tensões principais:

$$\sigma_1 = \sigma_I = 51.0 + \sqrt{(51.0)^2 + (0.326T)^2}$$

$$\sigma_2 = \sigma_{rr} = 0 \text{ MPa}$$

$$\sigma_3 = \sigma_{II} = 51.0 - \sqrt{(51.0)^2 + (0.326T)^2}$$

e portanto:

$$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = \sqrt{(51.0)^2 + (0.326T)^2}$$

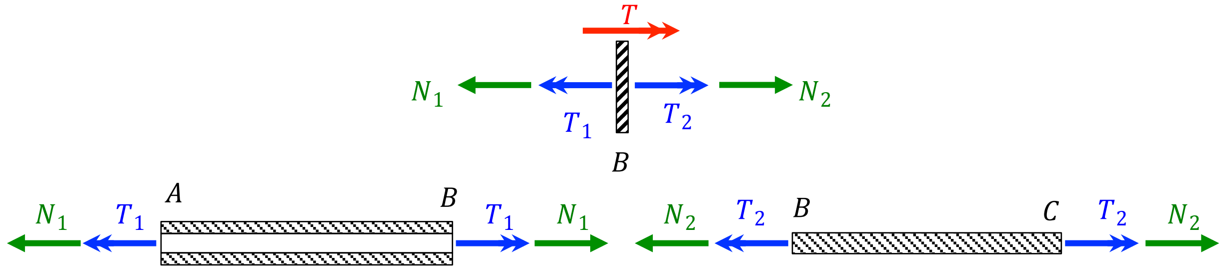
Para o cálculo do máximo torque admissível deve-se considerar que $\tau_{\max} < \tau_{\mathrm{y}}$.

Logo:

$$\sqrt{(51.0)^2 + (0.326T)^2} < 70.0 \text{ MPa} \quad \Rightarrow \boxed{T < 147 \text{ N}\cdot\text{m}}$$

Problema 4 (3,5 pontos)

(i) Equilíbrio:



$$N_1 = N_2 = N$$

$$T_1 - T_2 = T$$

(ii) Deslocamentos e rotações

$$u_B - u_A = \frac{NL_1}{E_1A_1} + \alpha_1 L_1 \Delta T = C_1 N + \Omega_1 \Delta T$$

$$u_C - u_B = \frac{NL_2}{E_2A_2} + \alpha_2 L_2 \Delta T = C_2 N + \Omega_2 \Delta T$$

$$\phi_B - \phi_A = \frac{T_1 L_1}{G_1 J_1} = \frac{T_1}{K_1}$$

$$\phi_C - \phi_B = \frac{T_2 L_2}{G_2 J_2} = \frac{T_2}{K_2}$$

onde $K_i = G_i J_i / L_i$, $C_i = L_i / E_i A_i$ e $\Omega_i = \alpha_i L_i$ $(i = 1,2)$.

(iii) Compatibilidade geométrica e condições de contorno

$$u_A = u_C = 0$$

$$\phi_A = \phi_C = 0$$

Portanto:

$$u_A = u_C = 0 \Rightarrow \; C_1 N + \Omega_1 \Delta T = -C_2 N - \Omega_2 \Delta T \; \Rightarrow N = -\frac{\Omega_1 + \Omega_2}{C_1 + C_2} \Delta T$$

$$\phi_A = \phi_C = 0 \Rightarrow \; \frac{T_1}{K_1} = -\frac{T_2}{K_2} \; \Rightarrow T_1 = \frac{K_1}{K_1 + K_2} T \; \text{ e } T_2 = -\frac{K_2}{K_1 + K_2} T$$

$K_1 = G_1 J_1 / L_1 = 1.70{\times}10^4 \text{ N} \cdot \text{m}$

$K_2 = G_2 J_2 / L_2 = 1.53{\times}10^4 \text{ N} \cdot \text{m}$

$C_1 = L_1 / E_1 A_1 = 3.82{\times}10^{-9} \text{ m/N}$

$C_2 = L_2 / E_2 A_2 = 2.04{\times}10^{-9} \text{ m/N}$

$\Omega_1 = \alpha_1 L_1 = 3.90{\times}10^{-6} \text{ m/°C}$

$\Omega_2 = \alpha_2 L_2 = 2.60{\times}10^{-6} \text{ m/°C}$

$N = -38.8 \text{ kN}$

$T_1 = 421 \text{ N} \cdot \text{m}$

$T_2 = -379 \text{ N} \cdot \text{m}$

Tubo:

$\sigma_{xx} = N_1 / A_1 = -98.9 \text{ MPa}$

$\max\{|\sigma_{x\theta}(r)|\} = |\sigma_{x\theta}(D_1/2)| = \left|\frac{D_1}{2} \frac{T_1}{J_1}\right| = 98.9 \text{ MPa}$

$$\sigma_m = \frac{\sigma_{xx} + \sigma_{x\theta}}{2} = -49.5 \text{ MPa}$$

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 111 \text{ MPa}$$

$$\sigma_I = \sigma_m + R = 61.1 \text{ MPa}$$

$$\sigma_{II} = \sigma_m - R = -160 \text{ MPa}$$

$\sigma_1 = \sigma_I = 61.1 \text{ MPa}$

$\sigma_2 = \sigma_{rr} = 0$

$\sigma_3 = \sigma_{II} = -160 \text{ MPa}$

$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 111 \text{ MPa}$

Barra:

$\sigma_{xx} = N_2 / A_2 = -79.1 \text{ MPa}$

$\max\{|\sigma_{x\theta}(r)|\} = |\sigma_{x\theta}(D_2/2)| = \left|-\frac{D_2}{2} \frac{T_2}{J_2}\right| = 124 \text{ MPa}$

$\sigma_m = \frac{\sigma_{xx} + \sigma_{x\theta}}{2} = -39.6 \text{ MPa}$

$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 130 \text{ MPa}$

$\sigma_I = \sigma_m + R = 90.2 \text{ MPa}$

$\sigma_{II} = \sigma_m - R = -169 \text{ MPa}$

$\sigma_1 = \sigma_I = 90.2 \text{ MPa}$

$\sigma_2 = \sigma_{rr} = 0$

$\sigma_3 = \sigma_{II} = -169 \text{ MPa}$

$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 130 \text{ MPa}$