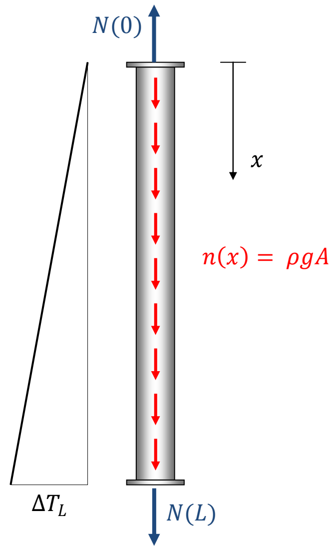
MECÂNICA DOS SÓLIDOS I

P1 – 27/04/2012 - Gabarito

Problema 1 (3,0 pontos).



$$\frac{dN}{dx} = -\rho g A \;\Rightarrow N(x) = -\rho g A x + c_1$$

$$\varepsilon = \frac{du}{dx} = -\frac{\rho g x}{E} + \frac{c_1}{EA} + \frac{\alpha \Delta T_L x}{L} \Rightarrow u(x) = -\frac{\rho g x^2}{2E} + \frac{c_1 x}{EA} + \frac{\alpha \Delta T_L x^2}{2L} + c_2$$

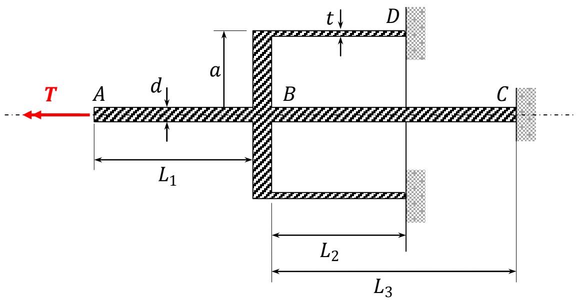
$$u(0) = 0 \;\Rightarrow\; c_2 = 0$$

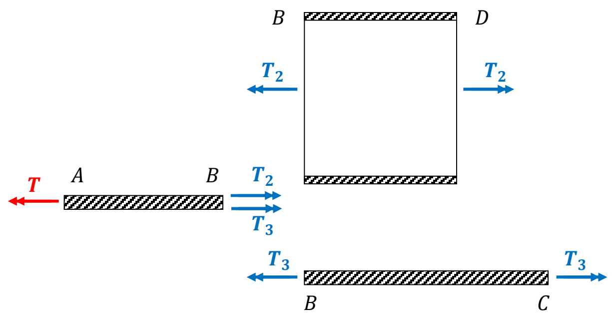
$$u(L) = 0 \;\Rightarrow\; c_1 = \frac{1}{2}\rho g A L - \alpha E A \frac{\Delta T_L}{2}$$

$$N(x) = \frac{1}{2}\rho g A L \left[1 - 2\left(\frac{x}{L}\right)\right] - \alpha E A \frac{\Delta T_L}{2}$$

$$N(0) = \frac{1}{2}\rho g A L - \alpha E A \frac{\Delta T_L}{2}\,, \quad N(L) = -\frac{1}{2}\rho g A L - \alpha E A \frac{\Delta T_L}{2}$$

Problema 2 (3,0 pontos).





$$\left.\begin{array}{c} T = T_2 + T_3 \\ \phi_A - \phi_B = T/k_1 \\ \phi_B - \phi_D = T_2/k_2 \\ \phi_B - \phi_C = T_3/k_3 \\ \phi_D = 0 \\ \phi_C = 0 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} T_2 = \dfrac{k_2}{k_2 + k_3} T \\ T_3 = \dfrac{k_3}{k_2 + k_3} T \\ \phi_B = \dfrac{T}{k_2 + k_3} \\ \phi_A = \dfrac{T}{k_1} + \phi_B \end{array}\right.$$

$$\phi_A = \left[\frac{1}{k_1} + \frac{1}{k_2 + k_3}\right] T = 2{,}72 \times 10^{-2}\ \text{rad}$$

$$J_1 = J_3 = \frac{\pi d^4}{32} = 1{,}57 \times 10^{-12}\ \text{m}^4$$

$$J_2 = \frac{\pi (2a)^3 t}{4} = 6{,}28 \times 10^{-9}\ \text{m}^4$$

$$k_1 = \frac{G J_1}{L_1} = 4{,}40\ \ \text{N} \cdot \text{m}$$

$$k_2 = \frac{G J_2}{L_2} = 17{,}6\ \times 10^3\ \text{N} \cdot \text{m}$$

$$k_3 = \frac{G J_3}{L_3} = 3{,}14\ \ \text{N} \cdot \text{m}$$

Problema 3 (3,5 pontos)

$$\left.\begin{array}{l} N = N_{Aço} + N_{Al} \\ \dfrac{\Delta L}{L} = \dfrac{N_{Aço}}{EA_{Aço}} = \dfrac{N_{AL}}{EA_{AL}} \end{array}\right\} \Rightarrow \left\{\begin{array}{l} N_{Aço} = \dfrac{EA_{Aço}}{EA_{Aço} + EA_{Al}} N = 46{,}9\ \text{kN} \\ N_{Al} = \dfrac{EA_{Al}}{EA_{Aço} + EA_{Al}} N = 13{,}1\ \text{kN} \end{array}\right.$$

$$A_{Aço} = \pi (D_{ext}^2 - D_{int}^2)/4 = 3{,}93 \times 10^{-4}\ \text{m}^2$$

$$A_{Al} = \pi D_{int}^2/4 = 3{,}14 \times 10^{-4}\ \text{m}^2$$

$$EA_{Aço} = 7{,}85 \times 10^7\ \text{N}$$

$$EA_{Al} = 2{,}20 \times 10^7\ \text{N}$$

$$\left.\begin{array}{l} T = T_{Aço} + T_{Al} \\ \dfrac{\Delta\phi}{L} = \dfrac{T_{Aço}}{GJ_{Aço}} = \dfrac{T_{AL}}{GJ_{AL}} \end{array}\right\} \Rightarrow \begin{cases} T_{Aço} = \dfrac{GJ_{Aço}}{GJ_{Aço} + GJ_{Al}} T = 186 \text{ N} \cdot \text{m} \\ T_{Al} = \dfrac{GJ_{Al}}{GJ_{Aço} + GJ_{Al}} T = 14{,}0 \text{ N} \cdot \text{m} \end{cases}$$

$$J_{Aço} = \pi(D_{ext}^4 - D_{int}^4)/32 = 6{,}38 \times 10^{-8} \text{ m}^4$$
$$J_{Al} = \pi D_{int}^4/32 = 1{,}57 \times 10^{-8} \text{ m}^4$$
$$GJ_{Aço} = 5{,}11 \times 10^3 \text{ N} \cdot \text{m}^2$$
$$GJ_{Al} = 3{,}93 \times 10^2 \text{ N} \cdot \text{m}^2$$

Na superície externa do revestimento de aço carbono:

$$\sigma_{xx} = N_{Aço}/A_{Aço} = 119 \text{ MPa}$$
$$\sigma_{xz} = (D_{ext}/2)\, T_{Aço}/J_{Aço} = 43{,}7 \text{ MPa}$$
$$\sigma_{zz} = 0$$

Logo:

$$\sigma_m = (\sigma_{xx} + \sigma_{zz})/2 = 59{,}7 \text{ MPa}$$
$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \sigma_{xz}^2} = 84{,}4 \text{ MPa}$$

$$\sigma_1 = \sigma_I = \sigma_m + R = 144 \text{ MPa}$$
$$\sigma_2 = 0$$
$$\sigma_3 = \sigma_{II} = \sigma_m - R = -24{,}7 \text{ MPa}$$

$$\tau_{\max} = (\sigma_1 - \sigma_3)/2 = 84{,}4 \text{ MPa}$$