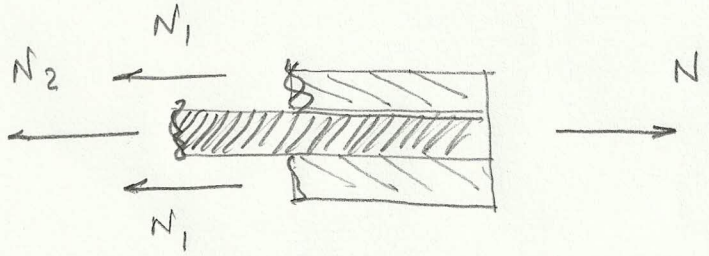
PROBLEMA 1



$$2N_1 + N_2 = N$$

$$\frac{\Delta L}{L} = \frac{N_1}{E_1 A_1} + \alpha_1 \Delta T = \frac{N_2}{E_2 A_2} + \alpha_2 \Delta T$$

$$N_2 = \frac{E_2 A_2}{E_1 A_1} N_1 + E_2 A_2 (\alpha_1 - \alpha_2) \Delta T$$

$$2N_1 + \frac{E_2 A_2}{E_1 A_1} N_1 + E_2 A_2 (\alpha_1 - \alpha_2) \Delta T = N$$

$$N_1 = \frac{E_1 A_1}{2E_1 A_1 + E_2 A_2} N - \frac{E_1 A_1 E_2 A_2}{2E_1 A_1 + E_2 A_2} (\alpha_1 - \alpha_2) \Delta T$$

$$\frac{\Delta L}{L} = \frac{N}{2E_1 A_1 + E_2 A_2} + \left[-\frac{E_2 A_2}{2E_1 A_1 + E_2 A_2} (\alpha_1 - \alpha_2) + \alpha_1 \right] \Delta T$$

$$= \frac{N}{2E_1 A_1 + E_2 A_2} + \left[\frac{2E_1 A_1 \alpha_1 + E_2 A_2 \alpha_2 - E_2 A_2 \alpha_1 + E_2 A_2 \alpha_2}{2E_1 A_1 + E_2 A_2} \right] \Delta T$$

$$= \frac{N}{2E_1 A_1 + E_2 A_2} + \frac{2E_1 A_1 \alpha_1 + E_2 A_2 \alpha_2}{2E_1 A_1 + E_2 A_2} \Delta T$$

$$E_{eq} = \frac{2E_1 A_1 + E_2 A_2}{A} = \frac{2E_1 t_1 + E_2 t_2}{t}$$

$$\alpha_{eq} = \frac{2E_1 t_1 \alpha_1 + E_2 t_2 \alpha_2}{2E_1 t_1 + E_2 t_2}$$

PROBLEMA 2

$$n(x) = -\rho g A$$

$$\frac{dN}{dx} = \rho g A \Rightarrow N(x) = \rho g A x + C_1$$

$$\frac{du}{dx} = \frac{N}{EA} = \frac{\rho g A}{EA} x + \frac{C_1}{EA}$$

$$u(x) = \frac{\rho g x^2}{2E} + \frac{C_1 x}{EA} + C_2$$

$$N(0) = k\,u(0) \Rightarrow C_1 = K C_2$$

Logo

$$u(x) = \frac{\rho g x^2}{2E} + \frac{C_1 x}{EA} + \frac{C_1}{K}$$

$$N(H) = -K\,u(H)$$

$$\rho g A H + C_1 = -\frac{K \rho g H^2}{2E} - \frac{KH}{EA} C_1 - C_1$$

$$\left(2 + \frac{KH}{EA}\right) C_1 = -\rho g A H - \frac{K \rho g H^2}{2E}$$

$$= -\frac{\rho g A H}{2}\left(2 + \frac{KH}{EA}\right)$$

$$C_1 = -\rho g A H/2$$

$$u(x) = \frac{\rho g x^2}{2E} - \frac{\rho g H x}{2E} - \frac{\rho g A H}{2K}$$

$$N(x) = \rho g A x - \frac{\rho g A H}{2}$$

ou

$$u(x) = -\frac{\rho g H^2}{2E}\left(\frac{x}{H}\right)\left[1 - \left(\frac{x}{H}\right)\right] - \frac{\rho g A H}{2K}$$

$$N(x) = -\frac{\rho g A H}{2}\left[1 - 2\left(\frac{x}{H}\right)\right]$$

Portanto

$$u(0) = u(H) = -\rho g A H / 2K$$

$$N(0) = -N(H) = -\rho g A H / 2$$

Problema 3

$$T = T_{al} + T_{aço}$$
$$N = N_{al} + N_{aço}$$

$$\frac{\Delta\phi}{L} = \frac{T_{al}}{G_{al} J_{al}} = \frac{T_{aço}}{G_{aço} J_{aço}}$$

$$\frac{\Delta L}{L} = \frac{N_{al}}{E_{al} A_{al}} = \frac{N_{aço}}{E_{aço} A_{aço}}$$

$$A_{aço} = \frac{\pi (D^2 - d^2)}{4}$$
$$A_{al} = \pi d^2/4$$
$$J_{Aço} = \frac{\pi (D^4 - d^4)}{32}$$
$$J_{al} = \pi d^4/32$$

Logo:

(a) $$N_{aço} = \frac{E_{aço} A_{aço}}{E_{aço} A_{aço} + E_{al} A_{al}} N \quad , \quad N_{al} = \frac{E_{al} A_{al}}{E_{aço} A_{aço} + E_{al} A_{al}} N$$

$$T_{aço} = \frac{G_{aço} J_{aço}}{G_{aço} J_{aço} + G_{al} J_{al}} T \quad , \quad T_{al} = \frac{G_{al} J_{al}}{G_{aço} J_{aço} + G_{al} J_{al}} T$$

(b) $$\frac{\Delta\phi}{L} = \frac{T}{G_{aço} J_{aço} + G_{al} J_{al}} \quad , \quad \frac{\Delta L}{L} = \frac{N}{E_{aço} A_{aço} + E_{al} A_{al}}$$

(c) $$\sigma_{xx} = \frac{N_{aço}}{A_{aço}} = \frac{E_{aço}}{E_{aço} A_{aço} + E_{al} A_{al}} N$$

$$\sigma_{\theta x} = \frac{D}{2} \frac{T_{aço}}{J_{aço}} = \frac{D}{2} \frac{G_{aço}}{G_{aço} J_{aço} + G_{al} J_{al}} T$$

$\sigma_{xl} = 100$ MPa

$\sigma_{\theta x} = 81.0$ MPa

$\sigma_m = \frac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = 50.2$ MPa

$R = \sqrt{\left(\frac{\sigma_{xl} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 95.2$ MPa

$\sigma_I = \sigma_m + R = 145$ MPa

$\sigma_{II} = \sigma_m - R = -45.1$ MPa

$\sigma_1 = \sigma_I = 145$ MPa
$\sigma_2 = 0$
$\sigma_3 = \sigma_{II} = -45.1$ MPa

$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 95.2$ MPa

$A_{aso} = 1.37 \times 10^{-4}\ m^2$

$A_{al} = 1.77 \times 10^{-4}\ m^2$

$J_{aso} = 1.07 \times 10^{-8}\ m^2$

$J_{al} = 4.97 \times 10^{-9}\ m^2$

$N_{aso} = 13.8\ kN$

$N_{al} = 6.21\ kN$

$T_{aso} = 86.9\ N.m$

$T_{al} = 13.1\ N.m$

$\frac{\Delta b}{L} = 0.101\ rad/m$

$\frac{\Delta L}{L} = 502 \times 10^{-6}$