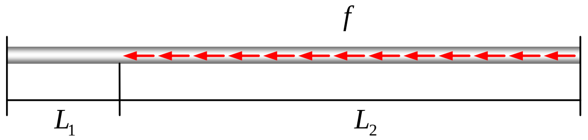
Problema 1 (3,0 pontos).



$$\frac{dN}{dx} + n(x) = 0\ ,\ n(x) = \begin{cases} 0, & 0 < x < L_1 \\ -f, & L_1 < x < L_1 + L_2 \end{cases}$$

$$\left.\begin{aligned} \frac{dN}{dx} &= 0, & 0 < x < L_1 \\ \frac{dN}{dx} &= f\,, & L_1 < x < L_1 + L_2 \end{aligned}\right\} \Rightarrow N(x) = \begin{cases} c_1, & 0 < x < L_1 \\ f\,x + d_1, & L_1 < x < L_1 + L_2 \end{cases}$$

$$N(L_1^-) = N(L_1^+) \quad \Rightarrow \quad d_1 = -f\,L_1 + c_1$$

$$\frac{du}{dx} = \frac{N(x)}{EA} + \alpha\,\Delta T \quad \Rightarrow \quad u(x) = \begin{cases} \dfrac{c_1 x}{EA} + \alpha\, x\Delta T + c_2, & 0 < x < L_1 \\ \dfrac{1}{EA}\left(-\dfrac{f(2L_1 x - x^2)}{2} + c_1 x \right) + \alpha\, x\Delta T + d_2, & L_1 < x < L_1 + L_2 \end{cases}$$

$$u(0) = 0 \Rightarrow c_2 = 0$$

$$u(L_1^-) = u(L_1^+) \Rightarrow \frac{c_1 L_1}{EA} + \alpha\, L_1 \Delta T = \frac{1}{EA}\left(-\frac{f\,L_1^2}{2} + c_1 L_1 \right) + \alpha\, L_1 \Delta T + d_2 \Rightarrow d_2 = \frac{f\,L_1^2}{2\,EA}$$

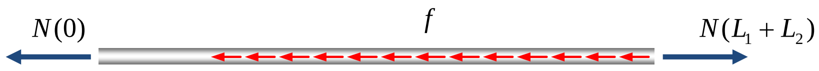
$$u(x) = \begin{cases} \dfrac{c_1 x}{EA} + \alpha\, x\Delta T\,, & 0 < x < L_1 \\ \dfrac{1}{EA}\left(-\dfrac{f(2L_1 x - x^2 - L_1^2)}{2} + c_1 x \right) + \alpha\, x\Delta T\,, & L_1 < x < L_1 + L_2 \end{cases}$$

$$u(L_1 + L_2) = 0 \Rightarrow -\frac{f\left(2L_1(L_1 + L_2) - (L_1 + L_2)^2 - L_1^2\right)}{2\,EA} + \frac{c_1(L_1 + L_2)}{EA} + \alpha\,(L_1 + L_2)\Delta T = 0$$

$$\Rightarrow \quad \frac{f\,L_2^2}{2\,EA} + \frac{c_1(L_1 + L_2)}{EA} + \alpha\,(L_1 + L_2)\Delta T = 0$$

$$\Rightarrow \quad c_1 = -\left[\frac{f\,L_2^2}{2(L_1 + L_2)} + \alpha EA\,\Delta T \right]$$

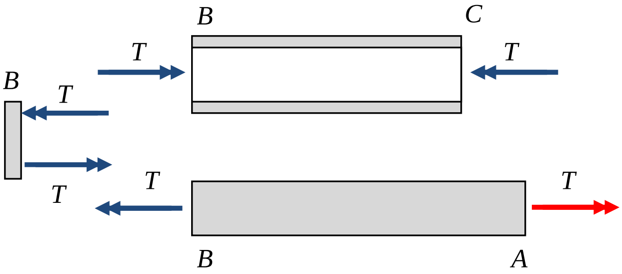
Resposta:



$$R_B = N(0) = c_1 = -\left[\frac{f\,L_2^2}{2(L_1 + L_2)} + \alpha EA\,\Delta T \right]$$

Problema 2 (3,0 pontos).

Equilíbrio:



Rotações:

$$\phi_A - \phi_B = \frac{TL_1}{GJ_1}, \quad J_1 = \frac{\pi D_1^4}{32}$$

$$\phi_C - \phi_B = -\frac{TL_2}{GJ_2}, \quad J_2 = \frac{\pi (D_1^4 - D_2^4)}{32}$$

$$\phi_C = 0 \Rightarrow \phi_B = \frac{32\, TL_2}{\pi G (D_1^4 - D_2^4)}$$

$$\phi_A = \frac{32\, TL_2}{\pi G (D_1^4 - D_2^4)} + \frac{32\, TL_1}{\pi G D_1^4}$$

(a) Mola de torção

$$K_T = \frac{T}{\phi_A} = \frac{\pi D_1^4 G}{32\, L_1} \left(1 + \frac{L_2 / L_1}{1 - (D_2 / D_1)^4} \right)^{-1}$$

(b) Tensões cisalhantes máximas

No eixo: $$\max \{\tau_1(r)\} = \tau_1(D_1/2) = \frac{D_1}{2} \frac{T}{\pi D_1^4 / 32} = \frac{16\, T}{\pi D_1^3}$$

Na camisa: $$\max \{\tau_2(r)\} = \tau_1(D_2/2) = \frac{D_2}{2} \frac{T}{\pi (D_2^4 - D_1^4)/32} = \frac{16\, T}{\pi (D_2^3 - (D_1^4 / D_2))}$$

$$\max \{\tau_1(r)\} = \max \{\tau 2_1(r)\} \Rightarrow \frac{16\, T}{\pi D_1^3} = \frac{16\, T}{\pi (D_2^3 - (D_1^4 / D_2))} \Rightarrow \frac{D_2^3}{D_1^3} - \frac{D_1}{D_2} = 1$$

Problema 3 (2,5 pontos).

$$\eta = \frac{V_F}{V_M} = \frac{A_F L}{A_M L} = \frac{A_F}{A_M}, \quad A = A_F + A_M = (1 + \eta) A_M$$

$$N_F + N_M = F$$

$$\delta_F = \frac{N_F L}{E_F A_F}$$

$$\delta_M = \frac{N_M L}{E_M A_M}$$

$$\Delta L = \delta_M = \delta_F \Rightarrow N_F = \frac{E_F A_F}{E_M A_M} N_M = \eta \frac{E_F}{E_M} N_M$$

$$N_M \left(1 + \eta \frac{E_F}{E_M} \right) = F \Rightarrow N_M = \frac{E_M}{E_M + \eta E_F} F$$

$$\Delta L = \frac{N_M L}{E_M A_M} = \frac{FL}{(E_M + \eta E_F) A_M} = \frac{1 + \eta}{E_M + \eta E_F} \frac{FL}{A}$$

Logo

$$\frac{\Delta L}{L} = \frac{FL}{E_C A} \Rightarrow E_C = \frac{E_M + \eta E_F}{1 + \eta}$$

Problema 4 (1,5 pontos).

$\sigma_{xx} = 100\,MPa$ $\sigma_{xz} = 20\,MPa$ $\sigma_{zz} = -30\,MPa$ $\sigma_{yy} = -50\,MPa$	**Plano *xz*** $\sigma_m = \frac{\sigma_{xx} + \sigma_{zz}}{2} = 35\,MPa$ $R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2}\right)^2 + \sigma_{xz}^2} = 68{,}0\,MPa$	$\sigma_I = \sigma_m + R = 103\,MPa$ $\sigma_{II} = \sigma_m - R = -33{,}0\,MPa$ $\sigma_1 = \sigma_I = 103\,MPa$ $\sigma_2 = \sigma_{II} = -33{,}0\,MPa$ $\sigma_3 = \sigma_{yy} = -50\,MPa$
$\tau_{\max} = \frac{\sigma_1 - \sigma_3}{2} = 76{,}5\,MPa$		