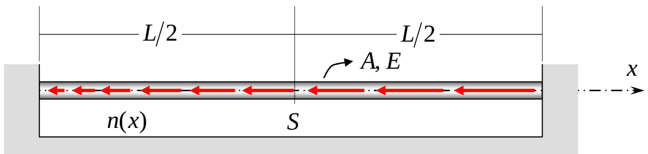
Problema 1 (3,0 pontos).



$$n(x) = -F\frac{x}{L}, \quad 0 < x < L$$

$$\frac{dN}{dx} - F\frac{x}{L} = 0 \quad \Rightarrow \quad N(x) = \frac{FL}{2}\left(\frac{x}{L}\right)^2 + c_1$$

$$\frac{du}{dx} = \frac{N(x)}{EA} \quad \Rightarrow \quad u(x) = \frac{FL^2}{6EA}\left(\frac{x}{L}\right)^3 + c_1\frac{L}{EA}\left(\frac{x}{L}\right) + c_2$$

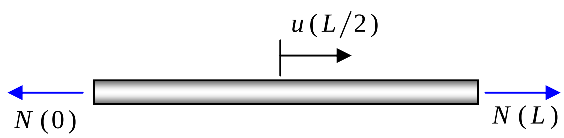
$$u(0) = 0 \quad \Rightarrow \quad c_2 = 0$$

$$u(L) = 0 \quad \Rightarrow \quad c_1 = -\frac{FL}{6}$$

$$u(x) = \frac{FL^2}{6EA}\left(\frac{x}{L}\right)\left[\left(\frac{x}{L}\right)^2 - 1\right]$$

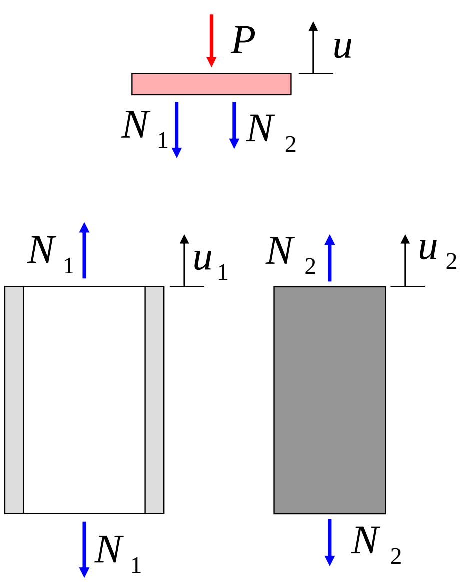
$$N(x) = \frac{FL}{6}\left[3\left(\frac{x}{L}\right)^2 - 1\right]$$

Respostas:



$$N(0) = -FL/6, \quad N(L) = FL/3, \quad u(L/2) = -FL^2/16\,EA$$

Problema 2 (2,5 pontos)



(i) Equilíbrio

$$N_1 + N_2 = -P$$

(ii) Deslocamentos

$$u_1 = \frac{L}{E_1A_1}N_1 + \alpha_1 L\,\Delta T$$

$$u_2 = \frac{L}{E_2A_2}N_2 + \alpha_2 L\,\Delta T$$

(iii) Compatibilidade de Deslocamentos:

$$u = u_1 = u_2$$

Logo, definindo-se: $C_i = \dfrac{L}{E_i A_i}$ e $\Omega_i = \alpha_i L \;\; (i = 1,2)$

$$u_1 = u_2 \quad \Rightarrow \quad C_1 N_1 + \Omega_1 \Delta T = C_2 N_2 + \Omega_2 \Delta T$$

Assim, considerando-se que $N_2 = -N_1 - P$:

$$C_1 N_1 + \Omega_1 \Delta T = -C_2 (N_1 + P) + \Omega_2 \Delta T$$

Rearranjando os termos, obtém-se:

$$N_1 = -\frac{C_2}{C_1 + C_2} P + \frac{\Omega_2 - \Omega_1}{C_1 + C_2} \Delta T$$

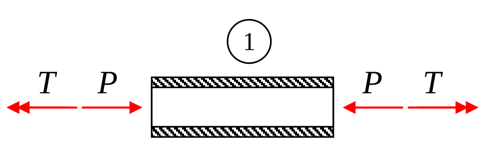
e portanto, considerando-se que $u = u_1 = C_1 N_1 + \Omega_1 \Delta T$:

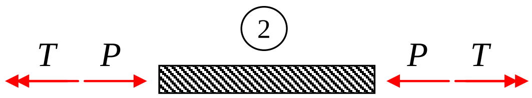
$$u = -\frac{C_1 C_2}{C_1 + C_2} P + \frac{C_1 \Omega_2 + C_2 \Omega_1}{C_1 + C_2} \Delta T$$

Problema 3 (1,5 pontos)

<table>
<tr>
<td>$\sigma_{xx} = 80\,MPa$
$\sigma_{xy} = 40\,MPa$
$\sigma_{yy} = 50\,MPa$
$\sigma_{zz} = -30\,MPa$</td>
<td>Plano xy
$\sigma_m = \dfrac{\sigma_{xx} + \sigma_{yy}}{2} = 65\,MPa$
$R = \sqrt{\left(\dfrac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2} = 42{,}7\,MPa$</td>
<td>$\sigma_I = \sigma_m + R = 108\,MPa$
$\sigma_{II} = \sigma_m - R = 22{,}3\,MPa$
$\sigma_1 = \sigma_I = 108\,MPa$
$\sigma_2 = \sigma_{II} = 22{,}3\,MPa$
$\sigma_3 = \sigma_{zz} = -30\,MPa$</td>
</tr>
<tr>
<td colspan="3">$\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2} = 68{,}9\,MPa$</td>
</tr>
</table>

Problema 4 (3,0 pontos).

<table>
<tr>
<td rowspan="3">
</td>
<td colspan="2">$\sigma_{xx} = -\dfrac{P}{A_1} = -56{,}6\,MPa$
$\max\{\sigma_{x\theta}\} = \dfrac{TD_1}{2J_1} = 55{,}2\,MPa$</td>
</tr>
<tr>
<td colspan="2">$\sigma_m = \dfrac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = -28{,}3\,MPa$
$R = \sqrt{\left(\dfrac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 62{,}0\,MPa$</td>
</tr>
<tr>
<td>$\sigma_I = \sigma_m + R = 5{,}45\,MPa$
$\sigma_{II} = \sigma_m - R = -119\,MPa$
$\sigma_1 = \sigma_I = 5{,}45\,MPa$
$\sigma_2 = 0\,MPa$
$\sigma_3 = \sigma_{II} = -119\,MPa$</td>
<td>$\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2} = 62{,}0\,MPa$</td>
</tr>
</table>

<table>
<tr>
<td rowspan="3">
</td>
<td colspan="2">$\sigma_{xx} = -\dfrac{P}{A_1} = -81{,}5\ MPa$
$\max\{\sigma_{x\theta}\} = \dfrac{TD_1}{2J_1} = 261 MPa$</td>
</tr>
<tr>
<td colspan="2">$\sigma_m = \dfrac{\sigma_{xx} + \sigma_{\theta\theta}}{2} = -40{,}7\ MPa$
$R = \sqrt{\left(\dfrac{\sigma_{xx} - \sigma_{\theta\theta}}{2}\right)^2 + \sigma_{x\theta}^2} = 264\ MPa$</td>
</tr>
<tr>
<td>$\sigma_I = \sigma_m + R = 182\ MPa$
$\sigma_{II} = \sigma_m - R = -345\ MPa$
$\sigma_1 = \sigma_I = 182\ MPa$
$\sigma_2 = 0\ MPa$
$\sigma_3 = \sigma_{II} = -345\ MPa$</td>
<td>$\tau_{\max} = \dfrac{\sigma_1 - \sigma_3}{2} = 264\ MPa$</td>
</tr>
</table>

A tensão cisalhante máxima ocorre na barra cilíndrica e seu valor é $\tau_{\max} = 264\ MPa$

Para os cálculos utilizou-se

$A_1 = 1{,}77 \times 10^{-4}\ m^2$

$A_2 = 1{,}23 \times 10^{-4}\ m^2$

$J_1 = 2{,}26 \times 10^{-8}\ m^4$

$J_2 = 2{,}40 \times 10^{-9}\ m^4$