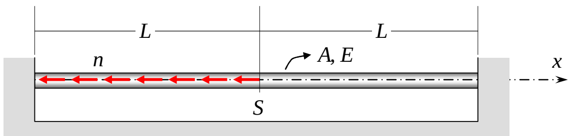
MECÂNICA DOS SÓLIDOS I

P1 – 20/04/2007

Gabarito

Problema 1 (3,5 pontos).



$$\frac{dN}{dx} + n(x) = 0\ ,\ n(x) = \begin{cases} -n, & 0 < x < L \\ 0, & L < x < 2L \end{cases}$$

$$\left.\begin{aligned} \frac{dN}{dx} - n = 0, \quad & 0 < x < L \\ \frac{dN}{dx} = 0, \quad & L < x < 0 \end{aligned}\right\} \Rightarrow N(x) = \begin{cases} nx + c_1, & 0 < x < L \\ c_2, & L < x < 0 \end{cases}$$

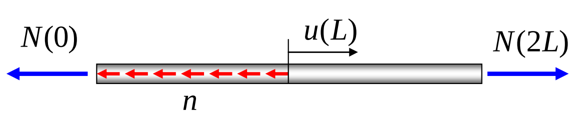
$$\frac{du}{dx} = \frac{N(x)}{EA} \quad \Rightarrow \quad u(x) = \begin{cases} \dfrac{1}{EA}\left(\dfrac{nx^2}{2} + c_1 x\right) + c_3, & 0 < x < L \\ \dfrac{c_2 x}{EA} + c_4, & L < x < 0 \end{cases}$$

$$\left.\begin{aligned} & u(0) = 0 \Rightarrow c_3 = 0 \\ & u(2L) = 0 \Rightarrow c_4 = -\frac{2Lc_2}{EA} \end{aligned}\right\} \Rightarrow u(x) = \begin{cases} \dfrac{nx^2 + 2c_1 x}{2EA}, & 0 < x < L \\ -\dfrac{c_2(2L - x)}{EA}, & L < x < 0 \end{cases}$$

$$\left.\begin{aligned} & u(L^-) = u(L^+) \ \Rightarrow \ \frac{nL^2 + 2c_1 L}{2EA} = -\frac{c_2 L}{EA} \\ & N(L^-) = N(L^+) \ \Rightarrow \ nx + c_1 = c_2 \end{aligned}\right\} \Rightarrow \begin{cases} c_1 = -3nL/4 \\ c_2 = nL/4 \end{cases}$$

$$u(x) = \begin{cases} -\dfrac{nL^2}{2EA}\left[\dfrac{3}{2}\left(\dfrac{x}{L}\right) - \left(\dfrac{x}{L}\right)^2\right], & 0 < x < L \\ -\dfrac{nL^2}{2EA}\left[1 - \dfrac{1}{2}\left(\dfrac{x}{L}\right)\right], & L < x < 2L \end{cases}$$

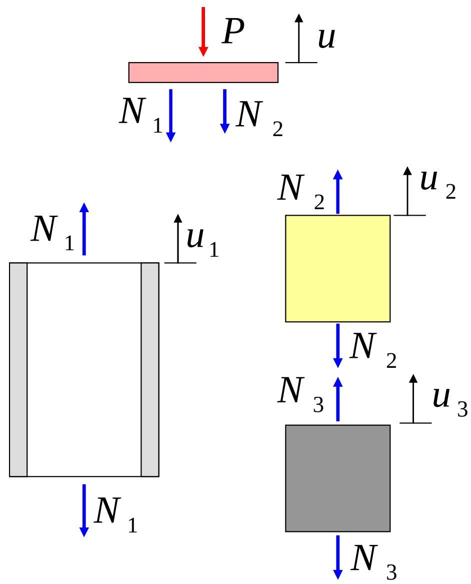
$$N(x) = \begin{cases} nL\left(\dfrac{x}{L} - \dfrac{3}{4}\right), & 0 < x < L \\ \dfrac{nL}{4}, & L < x < 2L \end{cases}$$



Respostas:

$$N(0) = -3nL/4$$
$$N(2L) = nL/4$$
$$u(L) = -nL^2/4EA$$

Problema 2 (3,0 pontos).



(i) Equilíbrio

$$N_1 + N_2 = -P$$

$$N_3 = N_2$$

(ii) Deslocamentos

$$u_1 = \frac{L_1}{E_1 A_1} N_1 + \alpha_1 L_1 \Delta T$$

$$u_2 = \frac{L_2}{E_2 A_2} N_2 + \alpha_2 L_2 \Delta T$$

$$u_3 = \frac{L_3}{E_3 A_3} N_3 + \alpha_3 L_3 \Delta T$$

(iii) Compatibilidade de Deslocamentos: $u = u_1 = u_2 + u_3$

Logo, definindo-se: $C_i = \dfrac{L_i}{E_i A_i}$ e $\Omega_i = \alpha_i L_i \ (i = 1,2,3)$

$$\left.\begin{array}{l} u_1 = u_2 + u_3 \\ N_2 = N_3 \end{array}\right\} \Rightarrow \quad C_1 N_1 + \Omega_1 \Delta T = (C_2 N_2 + \Omega_2 \Delta T) + (C_3 N_2 + \Omega_3 \Delta T)$$

Assim, considerando-se que $N_2 = -N_1 - P$:

$$C_1 N_1 + \Omega_1 \Delta T = -(C_2 + C_3)(N_1 + P) + (\Omega_2 + \Omega_3)\Delta T$$

Rearranjando os termos, obtém-se:

$$N_1 = -\frac{(C_2 + C_3)}{(C_1 + C_2 + C_3)} P + \frac{(\Omega_2 + \Omega_3 - \Omega_1)}{(C_1 + C_2 + C_3)} \Delta T$$

e portanto, considerando-se que $u = u_1 = C_1 N_1 + \Omega_1 \Delta T$:

$$u = -\frac{C_1 (C_2 + C_3)}{(C_1 + C_2 + C_3)} P + \left[\Omega_1 + \frac{(\Omega_2 + \Omega_3 - \Omega_1)}{(C_1 + C_2 + C_3)} \right] \Delta T$$

Finalmente:

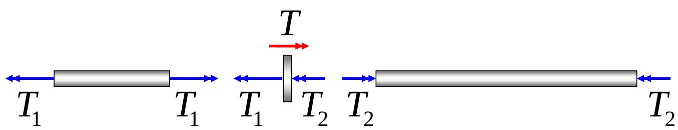
$$u = 0 \quad \Rightarrow \quad P = \left[\frac{\Omega_1 (C_1 + C_2 + C_3)}{C_1 (C_2 + C_3)} + \frac{(\Omega_2 + \Omega_3 - \Omega_1)}{(C_2 + C_3)} \right] \Delta T$$

$C_1 = 2{,}65 \times 10^{-10}$ m/N, $C_2 = 3{,}64 \times 10^{-10}$ m/N, $C_1 = 2{,}12 \times 10^{-10}$ m/N,
$\Omega_1 = 1{,}20 \times 10^{-6}$ m/°C, $\Omega_2 = 1{,}15 \times 10^{-6}$ m/°C e $\Omega_3 = 0{,}85 \times 10^{-6}$ m/°C.

$P = 200$ kN

Problema 3 (3,5 pontos).

Equilíbrio



$$T_1 + T_2 = P$$

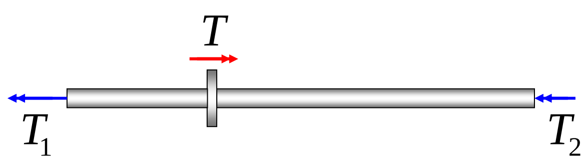
Rotações: $\phi_1 = \dfrac{T_1(L/3)}{GJ}$ e $\phi_2 = \dfrac{T_2(2L/3)}{GJ}$

Compatibilidade de Rotações: $\phi_1 = \phi_2$

$$\frac{T_1 L}{3GJ} = \frac{2T_2 L}{3GJ} \Rightarrow T_1 = 2T_2, \qquad \left.\begin{array}{l} T_1 = 2T_2 \\ T_1 + T_2 = T \end{array}\right\} \Rightarrow \begin{array}{l} T_1 = 2T/3 \\ T_2 = T/3 \end{array}$$

(i) Reações:

$T_1 = 2T/3 = 200$ N·m
$T_2 = T/3 = 100$ N·m



(ii) Rotação da Seção B: $\phi = \phi_1 = \phi_2 = \dfrac{2TL}{9GJ} = 0{,}042$ rad

(iii) Máxima tensão cisalhante: Ocorre em todos os pontos da superfície da barra entre as seções A e B.

$$\tau_{max} = \frac{D}{2}\frac{T_1}{J} = \frac{D}{2}\frac{T_1}{J} = \frac{32}{3}\frac{T}{\pi D^3} = 127 \text{ MPa}$$

(iv) Tensão normal devido ao carregamento axial causado pela variação de temperatura

$$\varepsilon = \frac{N}{EA} + \alpha\,\Delta T = 0 \Rightarrow N = -EA\,\alpha\,\Delta T = -37{,}7 \text{ kN}$$

$$\sigma = \frac{N}{A} = -120 \text{ MPa}$$

Ponto 1:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma & 0 & \tau \\ 0 & 0 & 0 \\ \tau & 0 & 0 \end{bmatrix} = \begin{bmatrix} -120 & 0 & 127 \\ 0 & 0 & 0 \\ 127 & 0 & 0 \end{bmatrix} \text{ MPa}$$

Ponto 2:

$$[\sigma] = \begin{bmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{xy} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{xz} & \sigma_{yz} & \sigma_{zz} \end{bmatrix} = \begin{bmatrix} \sigma & -\tau & 0 \\ -\tau & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -120 & -127 & 0 \\ -127 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ MPa}$$