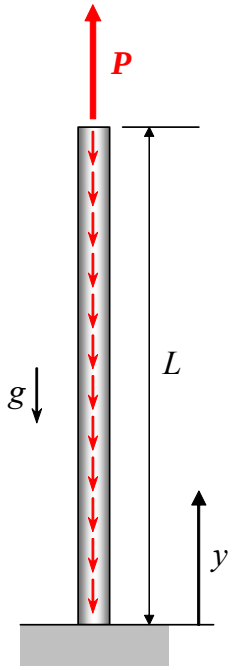
Problema 1.



$$q(y) = -\rho g A$$

$$\frac{dN}{dy} - \rho g A = 0 \quad \Rightarrow \quad N(y) = \rho g A y + c_1$$

$$N(L) = P \quad \Rightarrow \quad c_1 = P - \rho g A L \quad \Rightarrow \quad N(y) = P - \rho g A\,(L - y)$$

$$N(L/3) = 0 \quad \Rightarrow \quad P - \rho g A\,(L - L/3) = 0 \quad \Rightarrow \quad \boxed{P = \frac{2}{3}\rho g A L}$$

$$N(y) = \rho g A L \left(\frac{y}{L} - \frac{1}{3}\right)$$

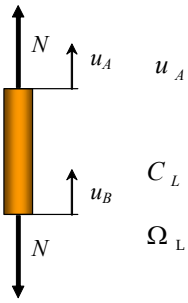
$$\frac{du}{dy} = \frac{N}{EA} \quad \Rightarrow \quad \frac{du}{dy} = \frac{\rho g L}{E}\left(\frac{y}{L} - \frac{1}{3}\right)$$

$$u(y) = \frac{\rho g L^2}{E}\left(\frac{y^2}{2L^2} - \frac{y}{3L} + c_2\right), \quad u(0) = 0 \quad \Rightarrow \quad c_2 = 0$$

$$\boxed{u(L/3) = -\frac{\rho g L^2}{18\,E}}$$

$$\boxed{P = 44{,}6 \text{ kN}} \qquad \boxed{u(L/3) = -12 \text{ mm}}$$

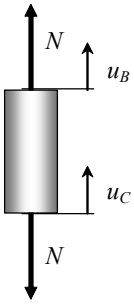
Problema 2.



$$u_A - u_B = \frac{L_L}{E_L A_L} N + \alpha_L L_L \Delta T$$

$$C_L = \frac{L_L}{E_L A_L} = 7{,}64 \times 10^{-9} \text{ m/N}$$

$$\Omega_L = \alpha_L L_L = 1{,}26 \times 10^{-6} \text{ m/ °C}$$



$$u_B - u_C = \frac{L_A}{E_A A_A} N + \alpha_A L_A \Delta T$$

$$C_A = \frac{L_A}{E_A A_A} = 1{,}82 \times 10^{-9} \text{ m/N}$$

$$\Omega_A = \alpha_A L_A = 0{,}92 \times 10^{-6} \text{ m/ °C}$$

$$u_C = 0 \Rightarrow u_B = C_A N + \Omega_A \Delta T \Rightarrow u_A = (C_A + C_L) N + (\Omega_A + \Omega_L) \Delta T$$

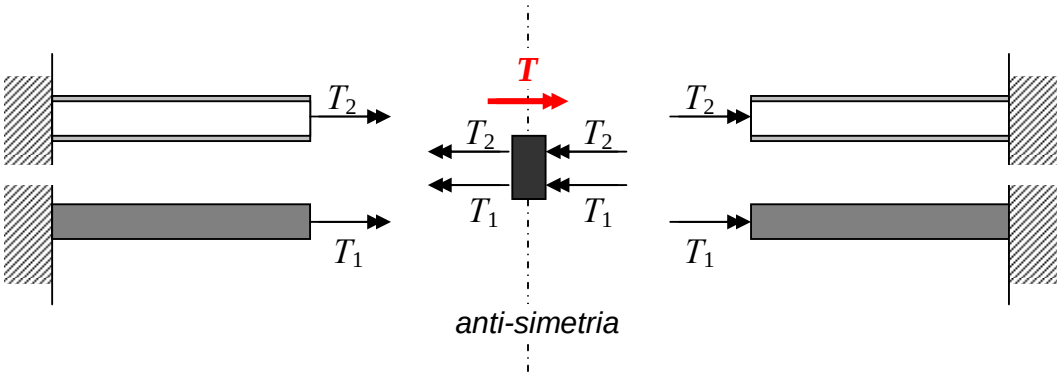
a) $\Delta T = 0, \quad N = -P, \quad u_A = -0{,}1 \text{ mm} \quad \Rightarrow \quad P = \dfrac{-u_A}{C_A + C_L} = 10{,}6 \text{ kN}$

b) $\sigma_L = -P/A_L = -135 \text{ MPa}, \quad \sigma_A = -P/A_A = -33{,}7 \text{ MPa}$

c) $u_B = -C_A P = -0{,}02 \text{ mm}$

d) $N = 0, \quad u_A = -0{,}1 \text{ mm} \quad \Rightarrow \quad \Delta T = \dfrac{u_A}{\Omega_A + \Omega_L} = -45{,}9\,°\text{C}$

Problema 3.



$$T_1 + T_2 = T/2$$

$$\phi_1 = \frac{T_1 L}{G_1 J_1} = \frac{T_1}{K_1}, \quad K_1 = \frac{G_1 J_1}{L} = 1{,}68 \times 10^5 \text{ N m/rad}$$

$$\phi_2 = \frac{T_2 L}{G_2 J_2} = \frac{T_2}{K_2}, \quad K_2 = \frac{G_2 J_2}{L} = 2{,}91 \times 10^5 \text{ N m/rad}$$

$$\phi_1 = \phi_2$$

$$T_1 = \frac{1}{2}\frac{K_1}{K_1 + K_2} T, \quad \text{e} \quad T_2 = \frac{1}{2}\frac{K_2}{K_1 + K_2} T$$

A máxima tensão cisalhante no eixo de aço é:

$$\max\left\{|\tau_1(r)|\right\} = \tau_1(D_1/2) = \frac{T_1 D_1}{2 J_1} = \frac{1}{4}\frac{K_1}{K_1 + K_2}\frac{D_1}{J_1} T = 7{,}45 \times 10^3 T$$

$$\max\left\{|\tau_1(r)|\right\} < 110 \times 10^6 \text{ Pa} \quad \Rightarrow \quad T_{\max} < \frac{110 \times 10^6}{7{,}45 \times 10^3} = 14{,}8 \text{ kN m}$$

A máxima tensão cisalhante no revestimento de alumínio é:

$$\max\left\{|\tau_2(r)|\right\} = \tau_2(D_2/2) = \frac{T_2 D_2}{2 J_2} = \frac{1}{4}\frac{K_2}{K_1 + K_2}\frac{D_2}{J_2} T = 4{,}77 \times 10^3 T$$

$$\max\left\{|\tau_2(r)|\right\} < 70 \times 10^6 \text{ Pa} \quad \Rightarrow \quad T_{\max} < \frac{70 \times 10^6}{4{,}77 \times 10^3} = 14{,}7 \text{ kN m}$$

$$\boxed{T_{\max} = 14{,}7 \text{ kN m}}$$